\[ 3x^2 + 2 = 29 \]
\[-2 \quad -2 \]
\[ 3x^2 = 27 \]
\[ \sqrt[3]{3} \quad \sqrt[3]{3} \]
\[ x^2 = 9 \]
\[ x = \pm 3 \]

\[
\begin{array}{c}
2 \\
\frac{2}{3} - \frac{1}{2} \\
\frac{4}{6} - \frac{3}{6} = \frac{1}{6} \\
\end{array}
\]

\[ a^2 + b^2 = c^2 \]
\[ 4^2 + x^2 = 5^2 \]
\[ 16 + x^2 = 25 \]
\[-16 \quad -16 \]
\[ x^2 = 9 \]
\[ x = 3 \]
4. What is sin(α), cos(α), tan(α)?
5. What is (3 + a)(4 + b)?
6. What is (c + f)^2?

Sohcahtoa

\[ \sin = \frac{3}{5} \]
\[ \cos = \frac{4}{5} \]
\[ \tan = \frac{3}{4} \]

FOIL

\[ 12 + 3b + 4a + ab \]

\[ (c + f)^2 = (c + f)(c + f) \]
\[ c^2 + cf + cf + f^2 = c^2 + 2cf + f^2 \]

\[ (x - 5)(x - 1) = 0 \]
\[ x - 5 = 0 \quad x = 5 \]
\[ x - 1 = 0 \quad x = 1 \]

\[ 5^2 - 6*5 + 5 = 0 \]
\[ 1^2 - 6*1 + 5 = 0 \]

Pythagorean theorem for distance

\[ a^2 + b^2 = c^2 \]
\[ 2^2 + 4^2 = c^2 \]
\[ 4 + 16 = 20 = c^2 \]
\[ c = \sqrt{20} \]

Distance graphically
Distance graphically

Distance algebraically
\[
\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}
\]
\[
(2,3) (4,7)
\]
\[
\sqrt{(7 - 3)^2 + (4 - 2)^2}
\]
\[
\sqrt{4^2 + 2^2} = \sqrt{20}
\]

HW
Find the distance graphically and algebraically between

(5,-7) (-1,3)
(2,4) (7,-6)
(-3,-8) (5,-2)
Quiz algebraically and graphically find the distance between -3, -8 and 5, -2

\[ \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \]
\[ \sqrt{(-2 - (-8))^2 + (5 - (-3))^2} \]
\[ \sqrt{(6)^2 + (8)^2} \]
\[ \sqrt{36 + 64} = \sqrt{100} = 10 \]

Slope:
\[ \Delta y \bigg/ \Delta x = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2 - (-8))}{(5 - (-3))} = \frac{6}{8} = \frac{3}{4} = .75 \]
HW
Read lesson 2 on handout pg 14
Find the slope graphically and algebraically between
(5, -7) (-1, 3)
(2, 4) (7, -6)
(-3, -8) (5, -2)
quiz Algebraically and graphically find the slope between (3,-7) and (-8,4). You need graph paper to get credit for graphically

\[
\frac{\Delta y}{\Delta x} = \frac{4 - (-7)}{-8 - 3} = \frac{11}{-11} = -1
\]

name independent

\[F(\text{year}) = \frac{250}{7}\]

dependent
In Problems 39–46, find the following for each function:

(a) \( f(0) \)  
(b) \( f(1) \)  
(c) \( f(-1) \)  
(d) \( f(-x) \)  
(e) \( -f(x) \)  
(f) \( f(x + 1) \)  
(g) \( f(2x) \)

39. \( f(x) = 3x^2 + 2x - 4 \)

40. \( f(x) = -2x^2 + x - 1 \)

41. \( f(x) = \frac{x}{x^2 + 1} \)

42. \( f(x) = \frac{2x + 1}{3x - 5} \)

43. \( f(x) = |x| + 4 \)

44. \( f(x) = \sqrt{x^2 + x} \)

45. \( f(x) = \frac{2x + 1}{3x - 5} \)

46. \( f(x) = \frac{2x + 1}{3x - 5} \)

39

a. \( f(0) = 3*0^2 + 2*0 - 4 = -4 \)
b. \( f(1) = 3*1^2 + 2*1 - 4 = 1 \)
c. \( f(-1) = 3*(-1)^2 + 2*(-1) - 4 = -3 \)
d. \( f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4 \)
e. \( -(3x^2 + 2x - 4) = -3x^2 - 2x + 4 \)
f. \( 3(x + 1)^2 + 2(x + 1) - 4 \)
   \[= 3(x^2 + 2x + 1) + 2x + 2 - 4 \]
   \[= 3x^2 + 6x + 3 + 2x + 2 - 4 = 3x^2 + 8x + 1 \]

1. Independent: people
Dependent: opinions
Why: to show how many reactions there were to Obama's inauguration
2. Independent: names
dependent: ages
Why: to show how people's age affected their reactions

Report 1 due 10/2
HW slope game
In Problems 39–46, find the following for each function:

(a) \( f(0) \)  \hspace{1cm} (b) \( f(1) \)  \hspace{1cm} (c) \( f(-1) \)  \hspace{1cm} (d) \( f(-x) \)  \hspace{1cm} (e) \( -f(x) \)  \hspace{1cm} (f) \( f(x + 1) \)  \hspace{1cm} (g) \( f(x) \)

39. \( f(x) = 3x^2 + 2x - 4 \)  
40. \( f(x) = -2x^2 + x - 1 \)

43. \( f(x) = |x| + 4 \)  
44. \( f(x) = \sqrt{x^2 + x} \)  
45. \( f(x) = \frac{2x + 1}{3x - 5} \)

a. \( 0/(0^2+1)=0 \)
b. \( 1/(1^2+1)=0.5 \)
c. \( (-1)/((-1)^2+1)=-0.5 \)
d. \( \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} \)
e. \( -\left( \frac{x}{x^2 + 1} \right) = \frac{-1}{1} \left( \frac{x}{x^2 + 1} \right) = \frac{-x}{x^2 + 1} \)

**Function:** \( -(1/2)x - 2 \)

**Points and Function Colors**
- red
- blue
- purple
- brown
- green

<table>
<thead>
<tr>
<th>X</th>
<th>Points</th>
<th>( 2x+1 )</th>
<th>( (1/2)x+2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td></td>
<td>-18.0</td>
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<td>0.5</td>
</tr>
<tr>
<td>-4</td>
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<td>-1</td>
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<tr>
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<td>-4.0</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td>5.0</td>
<td>-4.5</td>
</tr>
</tbody>
</table>
Graph $y=3x-5$
Handout pg. 27

1. **Independent: Index**  
   **Dependent: Price (Close)**  
   **Why:** to show increase or decrease in the market

2. **Independent: Bank**  
   **Dependent: Rate (interest)**  
   **Why:** so you can pick a bank with a good rate

**Formula for the line with slope 3 through 2,4**  
\( y = \text{slope} \times x + \text{intercept} \)
Algebraically
\[ y = \text{slope} \times x + \text{intercept} \]

\[ 4 = 3 \times 2 + \text{intercept} \]
\[ 4 = 6 + \text{intercept} \]
\[ -6 \quad 6 \]
\[ -2 = \text{intercept} \]
3, -5 with slope -2
y = mx + b
-5 = -2 * 3 + b
-5 = -6 + b
+6  +6
1 = b  y = -2x + 1

Find the formula for the line through
(-1, 1)  (3, 9)

Algebraically
\[ \frac{\Delta y}{\Delta x} = \frac{9 - 1}{3 - (-1)} = \frac{8}{4} = 2 \]

\[ 1 = 2 \times -1 + \text{int} \]

\[ 3 = \text{int} \]

\[ y = 2x + 3 \]

\[ (6, 8) \quad (-2, 4) \]

\[ \frac{4 - 8}{(-2 - 6)} = 0.5 \]

\[ 4 = \frac{1}{2} \times -2 + \text{int} \]

\[ 4 = -1 + \text{int} \]

\[ +1 \quad +1 \]

\[ 5 = \text{int} \]

\[ y = \frac{1}{2}x + 5 \]
HW
Find the formula for all the problems on pg 15 beginning of lesson 3
Algebraically and graphically
Graphically and algebraically find the formula for the line through (2,6) and (5,3)

From <https://mobile.twitter.com/Math4Sher/status/511635369935507456?p=v>

\[
\frac{\Delta y}{\Delta x} = \frac{3 - 6}{5 - 2} = -1 \\
3 = -1 \cdot 5 + b \\
3 = -5 + b \\
+5 + 5 \\
8 = b \\
y = -1x + 8
\]

Stock project 1 due 10/8

37. Car Rentals The cost $C$, in dollars, of renting a moving truck for a day is modeled by the function $C(x) = 0.25x + 35$. 
37. **Car Rentals**  The cost $C$, in dollars, of renting a moving truck for a day is modeled by the function $C(x) = 0.25x + 35$, where $x$ is the number of miles driven.

(a) What is the cost if you drive $x = 40$ miles?

(b) If the cost of renting the moving truck is $80, how many miles did you drive?

(c) Suppose that you want the cost to be no more than $100. What is the maximum number of miles that you can drive?

(d) What is the implied domain of $C$?

\[ a) \quad 0.25 \times 40 + 35 = 45 \]

\[ b) \quad 80 = 0.25x + 35 \]

\[ \begin{array}{c}
-35 \\
-35 \\
\end{array} \]

\[ 45 = 0.25x \]

\[ \begin{array}{c}
/.25 \\
/.25 \\
\end{array} \]

\[ 45/.25 = 180 = x \]

\[ c) \quad 100 \geq .25x + 35 \]

\[ \begin{array}{c}
-35 \\
-35 \\
\end{array} \]

\[ 65 \geq .25x \]

\[ \begin{array}{c}
/.25 \\
/.25 \\
\end{array} \]

\[ 65 \]

\[ \begin{array}{c}
.25 \\
\end{array} \]

\[ = 260 \geq x \]

\[ d) \quad \text{Non negative} \ [0, \infty) \]

HW pg 138-139 ex 38,45-50
\begin{align*}
\frac{(16086.41 - 15499.54)}{(5 - 1)} &= 146.7175 \text{ slope} \\
y &= \text{slope} \times x + \text{intercept} \\
y - \text{slope} \times x &= \text{intercept} \\
15499.54 - 146.7175 \times 1 &= 15352.8225 \text{ intercept} \\
146.7175 \text{month} + 15352.8225 \\
\end{align*}

\begin{align*}
\frac{(16717.17 - 16321.71)}{(11 - 8)} &= 131.82 \text{ slope} \\
16321.71 - 131.82 \times 8 &= 15267.15 \text{ intercept} \\
131.82 \text{month} + 15267.15 \\
\end{align*}

\begin{align*}
\frac{(16717.17 - 15499.54)}{(11 - 1)} &= 121.763 \text{ slope} \\
15499.54 - 121.763 \times 1 &= 15377.777 \text{ intercept} \\
121.763 \text{month} + 15377.777 \\
\end{align*}
<table>
<thead>
<tr>
<th>Date</th>
<th>Month</th>
<th>Price</th>
<th>Errors:</th>
<th>Errors:</th>
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<td>16526.204.8</td>
<td>16321.00</td>
<td>0.00</td>
<td>16351.30.17</td>
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<td>Week</td>
<td>Price 1</td>
<td>Price 2</td>
<td>Price 3</td>
<td>Price 4</td>
<td>Price 5</td>
<td></td>
</tr>
<tr>
<td>------------</td>
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<tr>
<td>2014-03</td>
<td>9</td>
<td>16457.66</td>
<td>16673.28</td>
<td>215.62</td>
<td>16453.53</td>
<td>4.13</td>
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<tr>
<td>2014-04</td>
<td>10</td>
<td>16580.84</td>
<td>16820.00</td>
<td>239.16</td>
<td>16585.35</td>
<td>4.51</td>
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<tr>
<td>2014-05</td>
<td>11</td>
<td>16717.17</td>
<td>16966.72</td>
<td>249.55</td>
<td>16717.17</td>
<td>0.00</td>
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<tr>
<td>2014-06</td>
<td>12</td>
<td>16826.6</td>
<td>17113.43</td>
<td>286.83</td>
<td>16848.99</td>
<td>22.39</td>
<td></td>
</tr>
</tbody>
</table>

Functions: (Edit function and click enter to graph)

From <http://matcmp.ncc.edu/sherd/Applets/Calclet/getStock2.php?stock=djia>

Dow Jones Industrial Average (^DJI)

* Follow

17,251.67

From <http://finance.yahoo.com/q?s=^DJI>
Current month is 15.5 but in your project you will use 16 and get the price at the beginning of October
146.7175*15.5+15352.8225 = 17626.9438
131.82*15.5+15267.15 = 17310.36
121.763*15.5+15377.777 = 17265.1035 closest.

The blue function (using the 1st 6 and the last 6) was the best predictor and generated a very close prediction. Though it only had the second best error total it still had a low error total so I chose the blue function.
42. Luxury Tax  In 2002, major league baseball signed a labor agreement with the players. In this agreement, any team whose payroll exceeded $136.5 million in 2006 had to pay a luxury tax of 40% (for second offenses). The linear function $T(p) = 0.40(p - 136.5)$ describes the luxury tax $T$ of a team whose payroll was $p$ (in millions of dollars).

**Source:** Major League Baseball

(a) What is the implied domain of this linear function?
(b) What was the luxury tax for the New York Yankees whose 2006 payroll was $171.1$ million?
(c) Graph the linear function.
(d) What is the payroll of a team that pays a luxury tax of $11.7$ million?
(e) Interpret the slope.

For every $1$ increase in payroll the tax increases by $0.40$
\[2x - 7 = -x + 5\]
\[+7 +7\]
\[2x = -x + 12\]
\[+x +x\]
\[3x = 12\]
\[/3 /3\]
\[x = 4\]

\[2 \times 4 - 7 = 1\]
\[-4 + 5 = 1\]
quiz Graphically and algebraically intersect $y = 2x+3$ and $y = 3x+6$

From <https://mobile.twitter.com/Math4Sher/status/514170277438431232?p=v>

$$2x+3=3x+6$$

-2x -2x

3=x+6

-6 -6

-3 =x

$2*-3+3=-3$

$3*-3+6=-3$
\[
\frac{(6-2)}{(7-3)} = 1
\]

\[y = mx + b\]

\[2 = 1 \times 3 + b\]

\[-3 - 3\]

\[-1 = b\]

\[y = x - 1\]

\[y = 2x + \text{int}\]

\[6 = 2 \times 2 + \text{int}\]

\[-4 - 4\]

\[2 = \text{int}\]

\[y = 2x + 2\]

\[2x + 2 = x - 1\]

\[-2 - 2\]

\[2x = x - 3\]
2x=x-3
-x  -x
x=-3
2*3+2=-4
3-1=-4

39. **Supply and Demand** Suppose that the quantity supplied \( S \) and quantity demanded \( D \) of T-shirts at a concert are given by the following functions:

\[
S(p) = -200 + 50p \\
D(p) = 1000 - 25p
\]

where \( p \) is the price of a T-shirt.

(a) Find the equilibrium price for T-shirts at this concert. What is the equilibrium quantity?

(b) Determine the prices for which quantity demanded is greater than quantity supplied.

(c) What do you think will eventually happen to the price of T-shirts if quantity demanded is greater than quantity supplied?

\[
-200+50p=1000-25p \\
+200+25p = +200+25p \\
75p=1200 \\
p = 1200/75 = 16 \text{ equilibrium price} \\
-200+50*16 = 600 \text{ equilibrium quantity} \\
1000-25*16 = 600
\]

When I charged $10 I sold 750 t-shirts but when I charged $20 I sold 500 t-shirts.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>750</td>
</tr>
<tr>
<td>20</td>
<td>500</td>
</tr>
</tbody>
</table>

\[
\frac{\Delta Q}{\Delta P} = \frac{500 - 750}{20 - 10} = -\frac{250}{10} = -25
\]

\[
500 = -25 \times 20 + \text{int} \\
500 = -500 + \text{int}
\]
\[ \Delta Q = \frac{500 - 750}{20 - 10} = -\frac{250}{10} = -25 \]

\[ \Delta P = \frac{500}{10} = -25 \]

\[ 500 = -25 \times 20 + \text{int} \]

\[ 500 = -500 + \text{int} \]

\[ +500 + 500 \]

\[ 1000 = \text{int} \]

\[ D(p) = -25p + 1000 \]

HW pg 138 prob 40
200 cats were sold at $40 each but at $50 150 cats were sold. What is the demand function. Charge what to sell 400 cats?

From <https://mobile.twitter.com/Math4Sher/status/516128400722120705?p=v>

\[
\frac{\Delta q}{\Delta p} = \frac{150 - 200}{50 - 40} = -\frac{50}{10} = -5
\]

\( q = \text{slope} \times p + \text{int} \)

\( 200 = -5 \times 40 + \text{int} \)

\( 200 = -200 + \text{int} \)

\( +200 \quad +200 \)

\( 400 = \text{int} \)

\( q = -5p + 400 \)

\( -5 \times 50 + 400 = 150 \)

\( 400 = -5p + 400 \)

\( p = 0 \)

The cheapest supplier of cats has 40 cats at $5 each, the next cheapest supplier of cats has 60 cats at $10 each, what is the supply function.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>40</td>
<td>5*40=200</td>
</tr>
<tr>
<td>+</td>
<td>60</td>
<td>10*60=600</td>
</tr>
</tbody>
</table>
\[
\frac{\Delta q}{\Delta p} = \frac{100 - 40}{8 - 5} = \frac{60}{3} = 20 \text{ slope}
\]

\[q = \text{slope} \times p + \text{int}\]

\[40 = 20 \times 5 + \text{int}\]

\[40 = 100 + \text{int}\]

\[-100 - 100\]

\[-60 = \text{int}\]

\[q = 20p - 60\]

\[20 \times 8 - 60 = 100\]

Equilibrium

\[20p - 60 = -5p + 400\]

\[-400 - 400\]

\[20p - 460 = -5p\]

\[+5p + 5p\]

\[25p - 460 = 0\]

\[-25p - 25p\]

\[-460 = -25p\]

\ытау{-25 -25\]

\[\frac{-460}{-25} = 18.4 = p \text{ equilibrium price}\]

\[1250 = -5 \times 18.4 + \text{int}\]

\[-5 \times 18.4 + 400 = 308\]

\[20 \times 18.4 - 60 = 308\]

Prob 8 pg 16

Pick unit (1/4 lb, lb, tons)

Lbs

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>( (2000-1250)/(3-4) = -750 \text{ slope} )</th>
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<tbody>
<tr>
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<td>5000/4 = 1250</td>
<td>( q = -750p + \text{int} )</td>
</tr>
<tr>
<td>4\times0.75 = 3</td>
<td>8000/4 = 2000</td>
<td></td>
</tr>
</tbody>
</table>
\[ q = -750p + \text{int} \]
\[ 1250 = -750 \cdot 4 + \text{int} \]
\[ 1250 + 750 \cdot 4 = 4250 = \text{int} \]

(a) \[ D(p) = -750p + 4250 \]
\[-750 \cdot 3 + 4250 = 2000 \]

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000/2000=0.5</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td>+</td>
<td>2000</td>
<td>1500</td>
</tr>
<tr>
<td>2500/4000=0.625</td>
<td>2000+2000=4000</td>
<td>1000+1500=2500</td>
</tr>
</tbody>
</table>

\[(4000-2000)/(0.625-0.5) = 16000.0 \text{ slope} \]
\[ 2000-16000 \times 0.5 = -6000 \text{ int} \]

\[ S(p) = 16000p - 6000 \]
\[ 16000 \times 0.625 - 6000 = 4000.0 \]

\[-750p + 4250 = 16000p - 6000 \]
\[ +750p + 6000 \quad +750p + 6000 \]
\[ 10250 = 16750p \]
\[ /16750 \quad /16750 \]
\[ 10250/16750 = 0.6119 \text{ Equilibrium price of lb} \]
Burger \[ 0.6119/4 = 0.153 \]
\[ 16000 \times 0.6119 - 6000 = 3790.4 \text{ equilibrium quantity} \]
\[-750 \times 0.6119 + 4250 = 3791.075 \]
Quiz
$1$ sells $375$  $1.25$ sells $325$  demand? cheapest $200$ for $150$ total  next cheapest $800$ for $750$ total  supply?  equilibrium price and quantity?

From <https://ncc.sln.suny.edu/webapps/blackboard/execute/announcement?method=edit&editMode=true&viewChoice=2&searchSelect=_22308_1&context=course&course_id=_22308_1&internalHandle=cp_announcements&announcementId=_51620_1>

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<tr>
<td>1</td>
<td>375</td>
<td>150</td>
</tr>
<tr>
<td>1.25</td>
<td>325</td>
<td>750</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
(325-375)/(1.25-1) &= -200 \\
375 &= -200*1 + \text{int} \\
&+200 \; +200 \\
575 &= \text{int} \\
D(p) &= -200p + 575 \\
-200*1.25 + 575 &= 325 \\
\end{align*}
\]

\[
\begin{array}{|c|c|c|}
\hline
P & Q & Total \\
\hline
150/200=0.75 & 200 & 150 \\
+ & 800 & 750 \\
900/1000=0.9 & 200+800=1,000 & 150+750=900 \\
\hline
\end{array}
\]

\[
(1000-200)/(0.9-0.75) = 5333.3333 \text{ slope} \\
1000 = 5333.3333*0.9 + \text{int} \\
1000-5333.3333*0.9 = -3800.0 \; =\text{int} \\
S(p) = 5333.3333p - 3800 \\
5333.3333*.75-3800=200.0 \checkmark
\]
5333.3333/.75-3800=200.0

-200p+575=5333.3333p-3800
+200p+3800 +200p+3800
4375=5533.3333p
/5533.3333 /5533.3333
4375/5533.3333 = 0.79  Equilibrium price
-200*.7907+575 = 416.86 equilibrium quantity
5333.33*.7907-3800 = 417.064

HW
Read 3.3
Slopes Report
Independent
Dependent
Increasing (or decreasing or flat)
Why

quiz $5 sells 7 $8 sells 1 demand? cheapest 2 for $3 each
next 8 for $5 each supply? equilibrium price&quant?

From [https://mobile.twitter.com/search?q=%23shermat111&s=typd]

<table>
<thead>
<tr>
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<th>Q</th>
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<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

(1-7)/(8-5) = -2 slope
1 = -2*8 + int
+16 +16
17 = int
D(p)= -2p + 17
D(5) = -2*5 + 17 = 7

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>3*2=6</td>
</tr>
<tr>
<td>+</td>
<td>8</td>
<td>5*8=40</td>
</tr>
<tr>
<td>46/10=4.6</td>
<td>2+8=10</td>
<td>6+40=46</td>
</tr>
</tbody>
</table>

(10-2)/(4.6-3)=5.0 slope
2 = 5*3 + int
-15 -15
-13 int
S(p)=5p-13
S(4.6) = 5*4.6-13 = 10

-2p+17=5p-13
+2p+13 +2p+13
30=7p
/7 /7
30/7 = 4.2857 Equilibrium price
-2*4.2857+17 = 8.4286 Equilibrium quantity
5*4.2857-13 = 8.4285
Quadratics

Vertex Form: $1(x-0)^2 + 0$
Normal Form: $1x^2 - 0x + 0$

Vertex Form: $1(x-0)^2 - 5$
Normal Form: $1x^2 - 0x - 5$
Vertex Form: $(x-4)^2 + 0$
Normal Form: $x^2 - 8x + 16$

Vertex Form: $(x-4)^2 - 5$
Normal Form: $x^2 - 8x + 11$

HW quadratics lesson
Pg 155 ex 11-18
Quiz graph $-1(x - 2)^2 + 4$

From [https://mobile.twitter.com/Math4Sher/status/519021459562565632?p=v](https://mobile.twitter.com/Math4Sher/status/519021459562565632?p=v)

$a(x-h)^2+k$

Algebraically

$-1(x + 2)^2 + 4 = 0$

$-4$  $-4$

$-1(x + 2)^2 = -4$

$\frac{-1}{(x + 2)^2} = 4$

$x + 2 = \pm 2$

-2  -2
\[ x = -2 \pm 2 \]
0 or -4

\[-1 \cdot (0+2)^2 + 4 = 0\]
\[-1 \cdot (-4+2)^2 + 4 = 0\]

Solve
\[ 2(x - 3)^2 - 3 = 0 \]
\[ \frac{2(x - 3)^2}{2} = \frac{3}{2} \]
\[ (x - 3)^2 = \frac{3}{2} \]
\[ x - 3 = \pm \sqrt{\frac{3}{2}} \]
\[ +3 \quad +3 \]
\[ x = 3 \pm \sqrt{\frac{3}{2}} \]

Graphically:

\[ 3 + \sqrt{\frac{3}{2}} = 4.224744871391589 \]
\[ 3 - \sqrt{\frac{3}{2}} = 1.775255128608411 \]
Slopes report
Pg 22 does not work
Pg 27
Databank
Independent: year
Dependent: price of the dow jones
Increasing
Why: it shows how the economy is doing

Pg 33
Independent: years
Dependent: percent black American below poverty line
Decreasing
Why: to show how black american poverty has decreased over the years

HW
Solve
\[2(x + 7)^2 - 9 = 0\]
\[-(x - 7)^2 + 8 = 0\]
Algebraically

\[ 2(x + 7)^2 - 9 = 0 \]

\[ +9 +9 \]

\[ 2(x + 7)^2 = 9 \]

\[ /2 /2 \]

\[ (x + 7)^2 = \frac{9}{2} \]

\[ x + 7 = \pm \sqrt{\frac{9}{2}} \]

\[ -7 -7 \]

\[ x = -7 \pm \sqrt{\frac{9}{2}} \]

\[ -7 + \sqrt{9/2} = -4.878679656440358 \]

\[ -7 - \sqrt{9/2} = -9.121320343559642 \]

\[ -(x - 7)^2 + 8 = 0 \]

\[ -8 -8 \]

\[ -(x - 7)^2 = -8 \]

\[ /-1 /-1 \]

\[ (x - 7)^2 = 8 \]

\[ x - 7 = \pm \sqrt{8} \]

\[ +7 +7 \]

\[ x = 7 \pm \sqrt{8} \]

\[ 7 + \sqrt{8} = 9.82842712474619 \]

\[ 7 - \sqrt{8} = 4.17157287525381 \]
\[ y = 2x^2 + 4x - 3 \]
\[ ax^2 + bx + c = a(x - h)^2 + k = a(x^2 - 2hx + h^2) + k \]
\[ = ax^2 - 2ahx + ah^2 + k \]

\[ b = -2ah \]
\[ \div -2a \div -2a \]
\[ \frac{b}{-2a} = h \]
\[ k = ah^2 + bh + c \]
\[ 2x^2 + 4x - 3 \]
\[ 4/(-2*2) = -1 = h \]
\[ 2*(-1)^2 + 4*-1 - 3 = -5 = k \]
\[ 2x^2 + 4x - 3 = 2(x + 1)^2 - 5 = 0 \]
\[ +5 \quad +5 \]

\[ 2(x + 1)^2 = 5 \]
\[ \div 2 \quad \div 2 \]
\[ (x + 1)^2 = \frac{5}{2} \]
\[ x + 1 = \pm \sqrt{\frac{5}{2}} \]
\[ -1 \quad -1 \]
\[ x = -1 \pm \sqrt{\frac{5}{2}} \]

HW
Graphically and algebraically solve:
\[ x^2 - 8x + 11 = 0 \]
\[ 3x^2 + 18x + 20 = 0 \]
1. Draw the points (3,9) and (-1,-7). Graphically and Algebraically find:

   a) The distance between the two points.
   
   b) The slope between the two points.
   
   c) The formula for the line between the two points.

25 pts.
2. Algebraically and Graphically: Find the intersection point of the line through (6,0,9,0) and (-2,0,5,0) and the line through (1,0,8,0) and (3,0,4,0).
25 pts.
3. Algebraically and Graphically: solve \(-2(x - 4)^2 + 6 = 0\)

25 pts.
4. A burger joint sold 500 quarter pound burgers when they charged $2.00 and 300 quarter pound burgers when they charged $2.50. They can get 400 quarter pounds of burger meat from one meat packing plant at $1.50 a quarter pound and another 600 quarter pounds at $2.00 a quarter pound. Show work:

   a) What is the demand function?
   
   b) What is the supply function?
   
   c) What is the equilibrium price?
   
   d) What is the equilibrium demand?

graphically
a) \(\sqrt{(-16)^2 + (-4)^2} = \sqrt{256 + 16} = \sqrt{272}\)
   
b) \(\frac{\Delta y}{\Delta x} = \frac{-16}{-4} = 4\)
   
c) \(y = 4x - 3\)

Algebraically
a) \(\sqrt{(-7 - 9)^2 + (-1 - 3)^2} = \sqrt{(-16)^2 + (-4)^2} = \sqrt{272}\)
   
b) \((-7-9)/(-1-3)=4\)
   
c) \(y = \text{slope} \times x + \text{intercept}\)
   
   \(9 = 4 \times 3 + \text{intercept}\)
   
   \(-12 = \text{intercept}\)
2.

Graphically
-4/4=-1
-4/2=-2
Algebraically
(5-9)/(-2-6)=-1 slope
y=slope*x+int
5=-1*2+int
-2 -2
3 = int
y=-1x+3
(4-8)/(3-1)=-2 slope
4=-2*3+int
+6 +6
10=int
y=-2x+10
-1x+3=-2x+10
+2x   +2x
x+3=10
  -3  -3
x=7
-1*7+3=-4 =y
-2*7+10=-4

3. Algebraically
-2(x - 4)^2 + 6 = 0
   -6   -6
-2(x - 4)^2 = -6
/ -2   / -2
(x - 4)^2 = 3
x - 4 = ±√3
  +4   +4
x = 4 ± √3
4+sqrt(3)=5.732050807568877
4-sqrt(3)=2.267949192431123

4. (300-500)/(2.5-2)=400 slope
500=-400*2+int
+800 +800
1200=int
4. |   |   | 500 = -400p + 1300 + 800 + 800 = 1300 = int a) \( D(p) = -400p + 1300 - 400 \times 2.5 + 1300 = 300 \)

\[
\begin{array}{c|c|c}
P & Q & Total \\
\hline
1.5 & 400 & 1.5 \times 400 = 600 \\
+ & 600 & 2 \times 600 = 1,200 \\
1800/1000 = 1.8 & 400 + 600 = 1,000 & 600 + 1200 = 1800 \\
\end{array}
\]

\[
(1000 - 400)/(1.8 - 1.5) = 2000.0 \text{ slope} \\
400 = 2000 \times 1.5 + \text{int} \\
400 - 2000 \times 1.5 = -2600 = \text{int} \\
b) \ S(p) = 2000p - 2600 \\
\quad 2000 \times 1.8 - 2600 = 1000 \checkmark \\
\]

\[
-400p + 1300 = 2000p - 2600 \\
+400p & 400p \\
1300 = 2400p - 2600 \\
+2600 & +2600 \\
3900 = 2400p \\
c) \ 3900/2400 = 1.625 \ \text{equilibrium price} \\
d) \ -400 \times 1.625 + 1300 = 650 \ \text{equilibrium demand} \\
\quad 2000 \times 1.625 - 2600 = 650 \checkmark 
\]
Quiz Graphically and algebraically solve:

\[ x^2 - 8x + 11 = 0 \]

\[ \frac{b}{-2a} = h \]

\[-8/(-2*1)=4 = h \]

\[ 4^2-8*4+11 = -5 = k \]

\[ (x - 4)^2 - 5 = 0 \]

\[ +5 \quad +5 \]

\[ (x - 4)^2 = 5 \]
\[ x - 4 = \pm \sqrt{5} \]
\[ +4 \quad +4 \]
\[ x = 4 \pm \sqrt{5} \]
\[ 4 + \sqrt{5} = 6.23606797749979 \]
\[ 4 - \sqrt{5} = 1.76393202250021 \]

\[ a(x - h)^2 + k = 0 \]
\[ -k \quad -k \]
\[ a(x - h)^2 = -k \]
\[ /a \quad /a \]
\[ (x - h)^2 = \frac{-k}{a} \]
\[ x - h = \pm \sqrt{\frac{-k}{a}} \]
\[ +h \quad +h \]
\[ x = h \pm \sqrt{\frac{-k}{a}} \]

Recipe for finding roots
\[ \frac{b}{-2a} = h \]
Plug h into quadratic for k
Graphically use a,h,k to draw quadratic
Circle where it hits the x axis
Algebraically roots are
\[ h \pm \sqrt{\frac{-k}{a}} \]
\[ 3x^2 + 18x + 20 = 0 \]
h = \frac{18}{-2 \times 3} = -3
K = 3 \times (-3)^2 + 18 \times -3 + 20 = -7

-3 \pm \sqrt{\frac{-7}{3}} = -3 \pm \frac{\sqrt{7}}{\sqrt{3}}

-3 + \sqrt{7/3} = -1.472474768348053
-3 - \sqrt{7/3} = -4.527525231651946
Quiz *Solve algebraically and graphically* $-2x^2 + 20x - 40 = 0$

$20/(-2*-2)=5 \Rightarrow h$

$-2*5^2 + 20*5 - 40 = 10 \Rightarrow k$

$5 \pm \sqrt{-\frac{10}{-2}} = 5 \pm \sqrt{5}$

$\sqrt{5} = 2.23606797749979$

$-2x^2 + 20x - 40 = y$

Intersects where with $4x-9$
\(-2x^2 + 20x - 40 = 4x - 9\)

\[-4x \quad -4x\]

\[-2x^2 + 16x - 40 = -9\]

\[+9 \quad +9\]

\[-2x^2 + 16x - 31 = 0 \quad \text{helper quadratic}\]

Roots of the helper are x values of intersection

\[16/(-2 \times -2) = 4 = h\]

\[-2 \times 4^2 + 16 \times 4 - 31 = 1 = k\]

\[4 \pm \sqrt{- \frac{1}{-2}} = x \text{ coordinates}\]

\[4 + \sqrt{\frac{1}{2}} = 4.707106781186548\]

\[4 \times 4.707 - 9 = 9.828 \ (4.707, 9.828)\]

\[-2 \times 4.707^2 + 20 \times 4.707 - 40 = 9.828 \text{ check}\]

\[4 - \sqrt{\frac{1}{2}} = 3.292893218813452\]

\[4 \times 3.2929 - 9 = 4.1716 \ (3.2929, 4.1716)\]
-2*3.2929^2+20*3.2929-40 = 4.1716 check

Recipe for quadratic line intersection
Graphically:
Graph quadratic
Graph line
Circle where they hit each other
Algebraically
Equate the formulas
Make helper quadratic = 0
Find roots of helper for x coordinates of answer
Plug roots into original formulas to get y coordinates of answer
Pg 16 lesson 6 1st problem
H = -4/(-2*1) = 2
K = 2^2-4*2+6 = 2

Graphically:
Graph quadratic
Graph line
Circle where they hit each other

Algebraically
Equate the formulas
Make helper quadratic = 0
Find roots of helper for x coordinates of answer
Plug roots into original formulas to get y coordinates of answer

Point and Function Colors
red blue purple brown
black green

Function: x^2-4x+6

<table>
<thead>
<tr>
<th>X</th>
<th>Points</th>
<th>2x+1</th>
<th>x^2-4x+6</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td></td>
<td>-19</td>
<td>146</td>
</tr>
<tr>
<td>-9.0</td>
<td></td>
<td>-17</td>
<td>123</td>
</tr>
<tr>
<td>-8.0</td>
<td></td>
<td>-15</td>
<td>102</td>
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<tr>
<td>-7.0</td>
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<td>-13</td>
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<td>-3.0</td>
<td>18</td>
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<td>-1.0</td>
<td>(-1,0.11)</td>
<td>-1.0</td>
<td>11</td>
</tr>
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<td>0.0</td>
<td>(0,6.0)</td>
<td>1.0</td>
<td>6.0</td>
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<tr>
<td>1.0</td>
<td>(1,3.0)</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>2.0</td>
<td>(2,2.0)</td>
<td>5.0</td>
<td>2.0</td>
</tr>
<tr>
<td>3.0</td>
<td>(3,3.0)</td>
<td>7.0</td>
<td>3.0</td>
</tr>
<tr>
<td>4.0</td>
<td>(4,6.0)</td>
<td>9.0</td>
<td>6.0</td>
</tr>
<tr>
<td>5.0</td>
<td>(5,11)</td>
<td>11.0</td>
<td>11</td>
</tr>
</tbody>
</table>

\[ x^2 - 4x + 6 = 2x + 1 \]
\[-2x - 1 \quad -2x - 1 \]
\[x^2 - 6x + 5 = 0\]  
Helper \( h = -6/(-2*1) = 3 \)

Helper \( k = 3^2 - 6*3 + 5 = -4 \)

\[
3 \pm \sqrt{-\frac{-4}{1}} = 3 \pm 2
\]

\[2*5+1 = 11 \quad (5,11)\]

\[5^2 - 4*5 + 6 = 11 \text{ check}\]

\[2*1+1 = 3 \quad (1,3)\]

\[1^2 - 4*1 + 6 = 3 \text{ check}\]

HW

2nd problem from lesson 6

Find the intersection of

\[2x^2 + 8x + 9 \text{ and the line through } -4,3 \text{ and } 1,-2\]
quiz Find the intersection of the line through -2,0 and -4,-8 and \( y = -2x^2 - 8x - 3 \). Make sure both intersections are on the graph.

From <https://mobile.twitter.com/Math4Sher/status/523952657392996352?p=v>

\[-8/(-2\times-2)=-2 =h\]
\[-2\times(-2)^2-8\times-2-3=5\]

Algebraically
\[(-8-0)/(-4-(-2)) = 4 \text{ slope}\]
\[0=4\times-2+\text{int}\]
\[ +8 +8 = \text{int} \]
\[ 4x + 8 = -2x^2 - 8x - 3 \]
\[-4x - 8 \quad -4x \quad -8 \]
\[ 0 = -2x^2 - 12x - 11 \text{ helper} \]
Helper h = \( \frac{-12}{-2*2} = -3 \)
Helper k = \(-2*(-3)^2-12*-3-11 = 7 \)

X values = \(-3 \pm \sqrt{-\frac{7}{-2}} \)
\[-3 + \sqrt{\frac{7}{2}} = -1.129171306613029 \]
\[4* -1.13 + 8 = 3.48 \ (-1.13, 3.48) \]
\[-2*(-1.13)^2-8*-1.13-3 = 3.4862 \text{ check} \]
\[-3 - \sqrt{\frac{7}{2}} = -4.87082869338697 \]
\[4* -4.87 + 8 = -11.48 \ (-4.87, -11.48) \]
\[-2*(-4.87)^2-8*-4.87-3=-11.4738 \text{ check} \]
quiz Find the formula for the quadratic through 2, 6, 3, 14, 5, 42

ax^2 + bx + c = y
2*2a+2b+c=6
3*3a+3b+c=14
5*5a+5b+c=42

4a+2b+c=6
9a+3b+c=14
25a+5b+c=42

25a+5b+c=42
-(4a+2b+c=6)
21a+3b=36
/3
7a+b=12

25a+5b+c=42
-(9a+3b+c=14)
16a+2b=28
/2
8a+b=14
\(-7a + b = 12\)
\(a = 2\)
\(7 \times 2 + b = 12\)
\(-14 \quad -14\)
\(b = -2\)

\(4a + 2b + c = 6\)
\(4 \times 2 + 2 \times -2 + c = 6\)
\(8 - 4 + c = 6\)
\(-4 \quad -4\)
\(c = 2\)
\(2x^2 - 2x + 2 = y\)
\(2 \times 2^2 - 2 \times 2 + 2 = 6\)
\(2 \times 3^2 - 2 \times 3 + 2 = 14\)
\(2 \times 5^2 - 2 \times 5 + 2 = 42\)
10a+b=51.455
-(7a+b=215.42)
3a = 300.035
/3 /3
a=100.01166
100.01166*1+484.6616*1 = -384.64994 +c=15499.54
15499.54+384.65=15884.19

Dow Jones Industrial Average (^DJI) - DJ
16,717.82 ↑256.50 (1.56%)
Way optimistic

\[-2.065 \times 64 + 171.055 \times 8 = 1,236.28 + c = 16321.71\]

\[16321.71 - 1,236.28 = 15085.43 = c\]
Quadratics through 3 Points

You can always find a quadratic of the form $ax^2 + bx + c$ that passes through 3 points. This page will guide you through the process of finding that quadratic.

Hint: Use three points to set up 3 equations.

<table>
<thead>
<tr>
<th>Point 1</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point 2</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point 3</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the points to construct 3 equations. $ax^2 + bx + c$ is the coefficient of $x$, and $c$ is the constant of $y$.

1. $a \cdot x_1^2 + b \cdot x_1 + c = y_1$
2. $a \cdot x_2^2 + b \cdot x_2 + c = y_2$
3. $a \cdot x_3^2 + b \cdot x_3 + c = y_3$

Once you construct the first system, solve it to evaluate $c$.

Then you construct the second system from the first to evaluate $b$.

Once you construct the second system from the first to evaluate $b$.

Now you construct the third system from the first to evaluate $a$.

Complete the next quadratic if the quadratic 7 and solve for $A$.

After plugging the value of $A$ into equation 7, you get the equation.

Write a system for $x = a$.
A little optimistic
## Quadratics through 3 Points

You can always find a quadratic of the form \(y = ax^2 + bx + c\) that passes through 3 points. This page will guide you through the process of finding that quadratic.

**Step 1:** Use the three points to set up 3 equations.

<table>
<thead>
<tr>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>(x_2)</td>
<td>(x_3)</td>
</tr>
<tr>
<td>(y_1)</td>
<td>(y_2)</td>
<td>(y_3)</td>
</tr>
</tbody>
</table>

**Step 2:** Solve the system of equations to find the values of \(a\), \(b\), and \(c\).

**Step 3:** Use the values of \(a\), \(b\), and \(c\) to write the quadratic equation.

**Example:**

Let's say we have the points:

- Point 1: \((1, 2)\)
- Point 2: \((2, 4)\)
- Point 3: \((3, 6)\)

The system of equations would be:

1. \(a(1)^2 + b(1) + c = 2\)
2. \(a(2)^2 + b(2) + c = 4\)
3. \(a(3)^2 + b(3) + c = 6\)

Solving this system will give us the values of \(a\), \(b\), and \(c\).
Somewhat optimistic
The green function from the last 6 months predicts the current price most accurately and has the least errors in the 12 months so $-2.065 \text{month}^2 + 171.055 \text{month} + 15085.43$ is the best.
Best linear function from project 1
121.763*month+15377.777

Best Quadratic function was
-2.065*month^2+171.055*month+15085.43

Best linear predicts for current price
121.763*16.7+15377.777=17411.2191

-2.065*16.7^2+171.055*16.7+15085.43 = 17366.1407 best quadratic predicts

Quadratic is slightly closer

The quadratic function prediction was a bit closer and it
had a bit less error on the 12 months so It was the best of the best.

Quadratics Report
Projectiles always have height as their dependent variable
Independent is always time or distance.
Why will be why is it a quadratic

Text pg 157 prob 87
Revenue is how much money a store or seller collects from buyers.

87. Maximizing Revenue Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is \( p \) dollars, the revenue \( R \) (in dollars) is

\[
R(p) = -4p^2 + 4000p
\]

What unit price should be established for the dryer to maximize revenue? What is the maximum revenue?

\[ h= \frac{4000}{(-2*-4)} = 500 \text{ price to maximize revenue}\]

\[ K = -4*500^2 + 4000*500 = 1000000 \text{ maximum revenue} \]

When the price was $700 we sold 1200 dryers but when we raised the price to $800 we sold 800 dryers.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>1200</td>
</tr>
<tr>
<td>800</td>
<td>800</td>
</tr>
</tbody>
</table>

\[
\frac{(800-1200)}{(800-700)} = -4 \text{ slope}\]

\[
1200 = -4*700 + \text{int}\]

\[
+2800 + 2800 \]

\[
4000 = \text{int}\]
\[ q = -4p + 4000 \]
\[ -4 \times 800 + 4000 = 800 \text{ check} \]

\[ R(p) = pq = p(-4p + 4000) = -4p^2 + 4000p \]

Each dryer cost us $300 rents and salaries cost us 200000
Cost function \( C(q) = 300q + 200000 \)
\( C(p) = 300(-4p + 4000) + 200000 = -1200p + 300 \times 4000 + 200000 \)
\( 300 \times 4000 + 200000 = 1400000 \)
\( C(p) = -1200p + 1400000 \)
Profit is Revenue - Cost
\( P(p) = R(p) - C(p) = -4p^2 + 4000p - (-1200p + 1400000) = -4p^2 + 5200p - 1400000 \)

\[ H = \frac{5200}{(-2 \times -4)} = 650 \text{ price to maximize profit} \]
\[ -4 \times 650^2 + 5200 \times 650 - 1400000 = 290000 \text{ maximum profit} \]
the American Mathematical Association of Two-Year Colleges (AMATYC) Student Mathematics League contest will be held on Tuesday, November 4, 2014, in room B-218 during Club Hour.

The level of the test is precalculus mathematics. The test contains 20 questions drawn from a standard syllabus in College Algebra and Trigonometry and may involve precalculus algebra, trigonometry, synthetic and analytic geometry, and probability; questions that are completely self-contained may be included as well. All questions are short-answer or multiple choice.

Calculators are allowed, including graphing calculators, provided they do not have a typewriter-like keyboard (for example, the TI-92) or a disk drive.

Quadratic newspaper report
Revenue quadratic
Dependent is either revenue or profit
Independent price (items sold) or quantity (sales)

Quiz
5 sell for $300 each at $250 10 sell.
Each costs $200, fixed cost $1000.
Demand function?
Maximum Revenue?
Cost Function?
Maximum Profit?

From https://ncc.sln.suny.edu/webapps/blackboard/execute/announcement?method=edit&editMode=true&viewChoice=2&searchSelect=22308_1&context=course&course_id=22308_1&internalHandler=cp_announcements&announcementId=52914_1

P | Q
---|---
300 | 5
250 | 10

\[(10-5)/(250-300) = -0.1 \text{ slope} \]

\[10 - -0.1 \times 250 = 35 \text{ intercept} \]

\[Q = -0.1p+35 \]

\[-0.1 \times 300+35 = 5 \text{ check} \]
\[ R(p) = pq = p(-0.1p + 35) = -0.1p^2 + 35p \]

\[ H = 35/(-2*-0.1) = 175 \text{ price that maximizes revenue} \]

\[ K = -0.1*175^2 + 35*175 = 3,062.5 \text{ maximum revenue} \]

\[ -0.1*175+35 = 17.5 \text{ sales to maximize revenue} \]

\[ C(q) = 200q+1000 \]

\[ C(p) = 200(-0.1p+35)+1000 = -20p+7000+1000=-20p+8000 \]

\[ P(p) = R(p)-C(p) = -0.1p^2 + 35p - (-20p + 8000) = -0.1p^2 + 55p - 8000 \]

\[ H = 55/(-2*-0.1) = 275 \text{ price that maximizes profit} \]

\[ K = -0.1*275^2+55*275-8000 = -437.5 \text{ maximum profit} \]

**Rational functions**

\[
\frac{2x}{x + 1}
\]

**Vertical asymptote**

Vertical asymptotes occur when the denominator = 0

\[
\frac{10}{x^2 - 6x + 5}
\]

\[ x^2 - 6x + 5 = 0 \]

\[ H = -6/(-2*1) = 3 \]

\[ K = 3^2-6*3+5 = -4 \]
Horizontal asymptotes
Degree of a polynomial is the largest power of x
5 = 5x^0
8x = 8x^1
If the denominator has a higher degree than the numerator then the horizontal asymptote is 0
If the denominator and numerator have the same degree
Then you get rid of all the lower degree terms
\[
\frac{2x}{x + 1} = \frac{2x}{x} = 2
\]
HW read 4.4
pg 225 prob 43 - 48
In Problems 43--54, find the vertical, horizontal, and oblique asymptotes, if any, of each rational function.

43. \( R(x) = \frac{3x}{x + 4} \)

44. \( R(x) = \frac{3x + 5}{x - 6} \)

45. \( H(x) = \frac{x^3 - 8}{x^2 - 5x + 6} \)

46. \( G(x) = \frac{x^3 + 1}{x^2 - 5x - 14} \)

47. \( T(x) = \frac{x^3}{x^4 - 1} \)

48. \( P(x) = \frac{4x^2}{x^3 - 1} \)

49. \( Q(x) = \frac{2x^2 - 5x - 12}{3x^2 - 11x - 4} \)

50. \( F(x) = \frac{x^2 + 6x + 5}{2x^2 + 7x + 5} \)

51. \( R(x) = \frac{6x^2 + 7x - 5}{3x + 5} \)

52. \( R(x) = \frac{8x^2 + 26x - 7}{4x - 1} \)

53. \( G(x) = \frac{x^1 - 1}{x^2 - x} \)

54. \( F(x) = \frac{x^4 - 16}{x^2 - 2x} \)

---

**Quiz prob 44,48**

44. \( x - 6 = 0 \)

\[ x = 6 \text{ Vertical asymptote} \]

Equal degrees

\[ \frac{3x}{x} = 3 \text{ horizontal asymptote} \]

48. \( x^3 - 1 = 0 \)

\[ x^3 = 1 \]

\[ x = 1 \text{ vertical asymptote} \]

Degree of the numerator = 2 < degree of denominator = 3

Horizontal asymptote is 0

---

When the numerator has a higher degree we use

Synthetic division

\[
\begin{array}{c|cccc}
58 & 1 & 0 & 0 & 0 \\
17 & & 85 & 150 & 136 & R 14 \\
\hline
58 & 14 & 17 & \end{array}
\]

\[ 1000 = 1 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 0 \times 10^0 \]
45. \( x^2 - 5x + 6 \div x^3 + 0x^2 + 0x - 8 \)
\[
\begin{align*}
x^3 & - 5x^2 + 6x \\
5x^2 & - 6x - 8 \\
5x^2 & - 25x + 30 \\
19x & - 38 \quad R
\end{align*}
\]
\[
(x - 2)(x^2 + 2x + 4) \\
(x - 2)(x - 3)
\]

51. \( 3x + 5 \div 6x^2 + 7x - 5 \)
\[
\begin{align*}
6x^2 & + 10x \\
-3x & - 5 \\
-3x & - 5 \\
R0
\end{align*}
\]

HW read 5.1
$$G(x) = \frac{x^3 + 1}{x^2 - 5x - 14} = x + 5 + \frac{39x + 71}{x^2 - 5x - 14}$$

$$x^2 - 5x - 14 = 0$$

$$h = -5/-2 = 2.5$$

$$k = 2.5^2 - 5*2.5 - 14 = -20.25$$

$$2.5 + \sqrt{-20.25/1} = 7$$

$$2.5 - \sqrt{-20.25/1} = -2$$

$$x^2 - 5x - 14 \mid x^3 + 0x^2 + 0x + 1$$

$$x^3 - 5x^2 - 14x$$

$$5x^2 + 14x + 1$$

$$5x^2 - 25x - 70$$

R 39x+71
Function composition

Stephanie(x) = 2x - 1
Lauren(x) = 5 - x

<table>
<thead>
<tr>
<th>X</th>
<th>stephanie ∘ lauren(x)</th>
<th>lauren ∘ stephanie(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>-11</td>
<td>-14</td>
</tr>
</tbody>
</table>

\[ f \circ g(x) = f(g(x)) \]

7. | x   | f(x) | g(x) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

(a) \((f \circ g)(1)\) 
(b) \((f \circ g)(-1)\) 
(c) \((g \circ f)(-1)\) 
(d) \((g \circ f)(0)\) 
(e) \((g \circ g)(-2)\) 
(f) \((f \circ f)(-1)\)

a) \(g(1) = 0\ f(0) = -1\)
b) \(g(-1) = 0\ f(0) = -1\)
c) \(f(-1) = -3\ g(-3) = 8\)
d) \(f(0) = -1\ g(-1) = 0\)
e) \(g(-2) = 3\ g(3) = 8\)
f) \(f(-1) = -3\ f(-3) = -7\)

\[ f \circ f(-3) \]
\[ f(-3) = -7 \]
\[ f(-7) \]
Cry
-3 is not in the domain of $f^\circ f$
$f^\circ g(6)$
g(6)
Cry

HW pg 256 ex 8,11-20
In Problems 11–20, for the given functions \( f \) and \( g \), find:

(a) \((f \circ g)(4)\)  
(b) \((g \circ f)(2)\)  
(c) \((f \circ f)(1)\)  
(d) \((g \circ g)(0)\)

11. \( f(x) = 2x; \quad g(x) = 3x^2 + 1 \)
   \( 13. f(x) = 4x^2 - 3; \quad g(x) = 3 - \frac{1}{2}x^3 \)

Quiz 256 prob 12

a. \( g(4) = 2 \times 4^2 - 1 = 31 \quad f(31) = 3 \times 31 + 2 = 95 \)
b. \( f(2) = 3 \times 2 + 2 = 8 \quad g(8) = 2 \times 8^2 - 1 = 127 \)
c. \( f(1) = 3 \times 1 + 2 = 5 \quad f(5) = 3 \times 5 + 2 = 17 \)
d. \( g(0) = 2 \times 0^2 - 1 = -1 \quad g(-1) = 2 \times (-1)^2 - 1 = 1 \)

In Problems 29–44, for the given functions \( f \) and \( g \), find:

(a) \( f \circ g \)  
(b) \( g \circ f \)  
(c) \( f \circ f \)  
(d) \( g \circ g \)

State the domain of each composite function.

29. \( f(x) = 2x + 3; \quad g(x) = 3x \)
31. \( f(x) = 3x + 1; \quad g(x) = x^2 \)
33. \( f(x) = x^2; \quad g(x) = x^2 + 4 \)
35. \( f(x) = \frac{3}{x-1}; \quad g(x) = \frac{2}{x} \)
37. \( f(x) = \frac{x}{x-1}; \quad g(x) = -\frac{4}{x} \)

29. Formula

a. \( f \circ g(x) = f\left( g(x) \right) = 2g(x) + 3 = 2(3x) + 3 = 6x + 3 \)
b. \( g \circ f(x) = g\left( f(x) \right) = 3f(x) = 3(2x + 3) = 6x + 9 \)
c. \( f^\circ f(x) = f(f(x)) = 2f(x) + 3 = 2(2x + 3) + 3 = 4x + 9 \)

d. \( g^\circ g(x) = g(g(x)) = 3g(x) = 3(3x) = 9x \)

35. .

a. Domain \( x \neq 0, x \neq 2 \)

\[
f^\circ g(x) = f(g(x)) = \frac{3}{g(x) - 1} = \frac{3}{\frac{2}{x} - \frac{1}{1}} = \frac{3}{\frac{2}{x} - \frac{1x}{1x}} = \frac{3}{\frac{2-x}{x}} = \frac{3}{\frac{2}{1}} * \frac{x}{2-x} = \frac{3x}{2-x}
\]

HW pg 257 30-44 read 5.2
Pg 257 ex 36

36. \( f(x) = \frac{1}{x + 3}; \quad g(x) = \frac{2}{x} \)

   a. Domain: \( x \neq 0, x \neq \frac{2}{3} \)
      
      \[ f^\circ g(x) = f(g(x)) = \frac{1}{g(x) + 3} = \frac{1}{\frac{2}{x} + 3} = \frac{1}{\frac{2 + 3x}{x}} = \frac{1}{-2 + 3x} \]
      
      \[ -\frac{2}{x} = -\frac{3}{1} \quad -2 = -3x \quad x = \frac{2}{3} \]

   b. Domain: \( x \neq -3 \)
      
      \[ g^\circ f(x) = g(f(x)) = -\frac{2}{f(x)} = -\frac{2}{\frac{1}{x + 3}} = -\frac{2}{1} \cdot \frac{x + 3}{1} = -2x - 6 \]

   c. Domain: \( x \neq -3, \quad x \neq -\frac{10}{3} \)
      
      \[ f^\circ f(x) = f(f(x)) = \frac{1}{f(x) + 3} = \frac{1}{\frac{1}{x + 3} + 3} = \frac{1}{\frac{1}{x + 3} + \frac{3(x + 3)}{1(x + 3)}} = \frac{1}{\frac{1 + 3x + 9}{x + 3}} \]
      
      \[ = \frac{x + 3}{10 + 3x} \]

   d. Domain: \( x \neq 0 \)
      
      \[ g^\circ g(x) = g(g(x)) = -\frac{2}{g(x)} = -\frac{2}{\frac{2}{x}} = -\frac{2}{2} \cdot \frac{x}{1} = x \]

43. \( f(x) = \frac{x - 5}{x + 1}; \quad g(x) = \frac{x + 2}{x - 3} \)

   a. Domain: \( x \neq 3, x \neq -\frac{1}{2} \)
      
      \[ f^\circ g(x) = f(g(x)) = \frac{g(x) - 5}{g(x) + 1} = \frac{\frac{x + 2}{x - 3} - 5}{\frac{x + 2}{x - 3} + 1} = \frac{x + 2 - 5(x - 3)}{x + 2 + x - 3} = \frac{x + 2 - 5x + 15}{2x - 1} = \frac{-4x + 17}{2x - 1} \]
      
      \[ = \frac{-4x + 17}{2x - 1} \]

   b. Domain: \( x \neq -1, x \neq -4 \)
      
      \[ \frac{x - 5}{x + 1} = \frac{3}{1} \quad x - 5 = 3(x + 1) = 3x + 3 \]
      
      \[ -x - 3 \quad -x - 3 \]
      
      \[ -8 = 2x \quad x = -4 \]
\[ g^o f(x) = g(f(x)) = \frac{f(x) + 2}{f(x) - 3} = \frac{x - 5 + 2}{x + 1} + \frac{3}{1} = \frac{x - 5 + 2(x + 1)}{x - 5 + 3(x + 1)} = \frac{x + 1}{x + 1} \] 

\[ = \frac{x - 5 + 2x + 2}{x - 5 - 3x - 3} = \frac{3x - 3}{x + 1} - \frac{2x - 8}{x + 1} = \frac{3x - 3}{-2x - 8} \]
44. \( f(x) = \frac{2x - 1}{x - 2}; \quad g(x) = \frac{x + 4}{2x - 5} \)

Domain: \( x \neq \frac{5}{2}, g(x) \neq 2 \) so \( x \neq \frac{14}{3} \)

\[
f^\circ g(x) = f(g(x)) = \frac{2g(x) - 1}{g(x) - 2} = \frac{2 \cdot \frac{x + 4}{2x - 5} - 1}{\frac{x + 4}{2x - 5} - 2} = \frac{2x + 8 - 1(2x - 5)}{x + 4 - 2(2x - 5)} = \frac{2x + 8 - 1(2x - 5)}{x + 4 - 2(2x - 5)} = \frac{13}{-3x + 14} = \frac{2x - 5}{-3x + 14}
\]

Inverse functions

Horizontal line test
Notation inverse of \( f(x) \) is \( f^{-1}(x) \)  
\[ f^{-1}(f(x)) = x \quad \text{also} \quad f(f^{-1}(y)) = y \]

Formula for \( f^{-1}(x) \)
\[ f(x) = 2x + 4 \]
\[ f(f^{-1}(x)) = x \]
\[ 2f^{-1}(x) + 4 = x \]
\[ \frac{2}{2} \]
\[ f^{-1}(x) + 2 = \frac{x}{2} \]
\[ -2 \quad -2 \]
\[ f^{-1}(x) = \frac{x}{2} - 2 \]

Quick check  \( f(1) = 2*1+4 = 6 \quad f^{-1}(6) = \frac{6}{2} - 2 = 1 \)

Complete check
\[ f^{-1}(f(x)) = \frac{f(x)}{2} - 2 = \frac{2x + 4}{2} - 2 \]
\[ = x + 2 - 2 = x \]

HW pg 268 ex 15-18  pg 269 ex 49-60
52. \( f(x) = 1 - 3x \)

\[ y = f^{-1}(x) \]
\[ f(y) = x \]
\[ 1 - 3y = x \]
\[ -1 - 1 \]
\[ -3y = x - 1 \]
\[ /-3 /-3 \]
\[ f^{-1}(x) = y = \frac{x - 1}{-3} \]
\[ f^{-1}(f(x)) = \frac{f(x) - 1}{-3} = \frac{\frac{1 - 3x - 1}{-3}}{-3} = \frac{-3x}{-3} = x \]

59. \( f(x) = \frac{1}{x - 2} \)

\[ f(y) = x \]
\[ \frac{1}{y - 2} = \frac{x}{1} \quad \text{cross multiply!} \]
\[ 1 = x(y - 2) = xy - 2x \]
\[ +2x +2x \]
\[ 1 + 2x = xy \]
\[ /x /x \]
\[ \frac{1 + 2x}{x} = y = f^{-1}(x) \]

quick check
\[ f(1) = \frac{1}{(1-2)} = -1 \]
\[ f^{-1}(-1) = \frac{1 + 2 \times -1}{-1} = \frac{-1}{-1} = 1 \]

**Complete check**

\[ f^{-1}(f(x)) = \frac{1+2f(x)}{f(x)} = \frac{\frac{1}{1+\frac{2}{x-2}}}{\frac{1}{x-2}} = \frac{(x-2)^1 + 2}{x-2} = \frac{x-2 + 2}{x-2} = \frac{x}{x-2} \]

\[ \frac{x}{x-2} \times \frac{x-2}{1} = \frac{x}{1} = x \]

63. \[ f(x) = \frac{3x}{x + 2} \]

\[ f(y) = x \]
\[ \frac{3y}{y + 2} = \frac{x}{1} \]

**Cross multiply!**

\[ x(y + 2) = 3y \]
\[ xy + 2x = 3y \]
\[ xy = -xy \]
\[ 2x = 3y - xy = y(3-x) \]
\[ \frac{2x}{3-x} = y = f^{-1}(x) \]

**Quick check**

\[ f(1) = \frac{3 \times 1}{(1+2)} = 1 \]
\[ f^{-1}(1) = \frac{2 \times 1}{3-1} = \frac{2}{2} = 1 \]
\[ f^{-1}(f(x)) = \frac{2f(x)}{3 - f(x)} = \frac{\frac{2}{3} \cdot \frac{3x}{x + 2}}{1 - \frac{3x}{x + 2}} = \frac{6x}{x + 2} \]

\[ = \frac{3(x + 2)}{1(x + 2)} - \frac{3x}{x + 2} = \frac{3x + 6 - 3x}{x + 2} = \frac{6x}{x + 2} \]

\[ = \frac{6x}{x + 2} \times \frac{x + 2}{6} = x \]

HW pg 369 ex 61-70 read 5.3
68. \( f(x) = \frac{2x - 3}{x + 4} \)

\[
y = f^{-1}(x) \\
f(y) = x \\
2y - 3 = \frac{x}{y + 4} = 1 \\
2y - 3 = x(y + 4) = xy + 4x \\
-xy \\
2y - xy - 3 = 4x \\
\quad + 3 \quad + 3 \\
y(2 - x) = 4x + 3 \\
/2 - x \quad /2 - x \\
\frac{f^{-1}(x)}{x} = y = \frac{4x + 3}{2 - x}
\]

Quick check
\[
f(-3) = \frac{2(-3) - 3}{(-3) + 4} = -9 \\
f^{-1}(-9) = \frac{4(-9) + 3}{2 - (-9)} = -3 \\
\]

\[
f^{-1}(f(x)) = \frac{4f(x) + 3}{2 - f(x)} = \frac{4 \cdot \frac{2x - 3}{x + 4} + 3}{2 - \frac{2x - 3}{x + 4}} = \frac{8x - 12 + 3}{2 - 2x - 3} = \frac{8x - 12 + 3x + 12}{2x + 8 - 2x + 3} = \frac{11x}{1(x + 4)}
\]

Exponentials
Laws of Exponents

If \( s, t, a, \) and \( b \) are real numbers with \( a > 0 \) and \( b > 0 \), then

\[
\begin{align*}
    a^s \cdot a^t &= a^{s+t} \\
    (a^s)^t &= a^{st} \\
    (ab)^s &= a^s \cdot b^s \\
    1^s &= 1 \\
    a^{-s} &= \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \\
    a^0 &= 1
\end{align*}
\]  

(1)

\[
\begin{align*}
    2^5 &= 1 \times 2 \times 2 \times 2 \times 2 \times 2 \\
    (2^4)^3 &= 1 \times 2^4 \times 2^4 \times 2^4 = 2^{12} \\
    2^{4^3} &= 2^{64}
\end{align*}
\]
$y = \langle b \rangle^{x}$

(coefficients
(y-intercept)

$10%$ increase
Red is a 10% decrease because .9 is 10% less than 1

2,6
4,24
24 = db^4
6 = db^2
\[ \frac{24}{6} = d = b^{4-2} = b^2 \]
4 = b^2  \hspace{1cm} \text{negative bases not allowed}
2 = b
6 = c2^2
/4  /4
1.5 = c
y = 1.5(2^x)
3.2
6.8
8 = cb^6
2 = cb^3
4 = b^3
4^{(1/3)} = 1.5874 = b
2 = c 1.5874^3
/1.5874^3 /1.5874^3
2 /1.5874^3 = 0.5
y = 0.5 * 1.5874^x
0.5 * 1.5874^6 = 8.0 check

-3,10
5,4
4 = cb^5
10 = cb^{-3}
.4 = b^8
.4^{(1/8)} = 0.8918
10 = c * 0.8918^{-3}
$10 / 0.8918^{-3} = 7.0925 = \text{c}$

$y = 7.0925 \times 0.8918^x$

$7.0925 \times 0.8918^5 = 4.0007 \text{ check}$
HW read 6.1 and 6.2
1. **30 Pts:** Graphically and algebraically find the intersection of the line through 1,0 and -1,8 and the quadratic \(-2x^2 + 12x - 20\)

2. **35 Pts:** Your shop will sell 1700 cones of grape ice cream for $1.50 a cone but if you charge $1.25 you will sell 1900 cones. You can make cones for $0.50 each and you have fixed costs of $360.00
   
   a. What is the demand function for cones?

   b. What is the revenue function for your shop?

   c. What is the price that maximizes revenue?

   d. What is the maximum revenue?

   e. What is the cost function for the shop?

   f. What the function that describes profit as a function of price?

   g. What is the price that maximizes profit?

   h. What is the maximum profit?

3. **35 pts** Consider \(f(x) = \frac{7-6x}{4+3x}\)

   a. (5pts) What are the vertical and horizontal asymptotes of \(f(x)\)

   b. (15pts) What is the formula for \(f^{-1}(x)\)

   c. (15pts) Check your formula by showing \(f^{-1}(f(x)) = x\)

---

**Show work on all problems**
1. Algebraically
   Slope = (0-8)/(1-1) = -4
   Intercept 0 - 4*1 = 4
   y=-4x+4
   -4x + 4 = -2x^2 + 12x - 20
   +4x-4 +4x - 4
   Helper quadratic 0 = -2x^2 + 16x - 24
   Helper h = 16/(-2 * -2) = 4
   Helper k = -2*4^2+16*4-24 = 8
   X values of the intercept 4 ± \sqrt{-8/-2}
   4+sqrt(-8/-2) = 6
   -4*6+4 = -20 (6,-20)
   -2*6^2+12*6-20=-20 check
   4-sqrt(-8/-2) = 2
   -4*2+4 = -4 (2,-4)
   -2*2^2+12*2-20 = -4 check
   Graphically

   ![Graphical representation of the line and parabola]

   h= 12/(-2*-2)=3
\[ k = -2 \times 3^2 + 12 \times 3 - 20 = -2 \]

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1700</td>
</tr>
<tr>
<td>1.25</td>
<td>1900</td>
</tr>
</tbody>
</table>

Slope \((1900-1700)/(1.25-1.5) = -800\)
Intercept \(1700 - (-800 \times 1.5) = 2900\)

\( a. \ q = -800p + 2900 \)
\[-800 \times 1.25 + 2900 = 1900 \text{ check}\]

\( b. \ R(p) = pq = p(-800p + 2900) = -800p^2 + 2900p \)
\( c. \ 2900/(-2 \times -800) = 1.8125 \text{ price that maximizes revenue}\)
\( d. \ -800 \times 1.81^2 + 2900 \times 1.81 = 2628.12 \text{ maximum revenue}\)

\( e. \ C(q) = 0.5q + 360\)
\( C(p) = 0.5(-800p + 2900) + 360 = -400p + 1450 + 360 = -400p + 1810\)

\( f. \ P(p) = R(p) - C(p) = -800p^2 + 2900p - (-400p + 1810)\)
\[= -800p^2 + 2900p + 400p - 1810\]
\[= -800p^2 + 3300p - 1810\]

\( g. \ 3300/(-2 \times -800) = 2.0625 \text{ price that maximizes profit}\)
\( h. \ -800 \times 2.06^2 + 3300 \times 2.06 - 1810 = 1593.12 \text{ maximum profit}\)

3.

\( a. \ \text{Vertical} \ 4 + 3x = 0 \quad 3x = -4 \quad x = -4/3\)

\[ \text{horizontal} \quad \frac{-6x}{3x} = -2 \]

\( b. \ f(y) = x\)
\[ \frac{7 - 6y}{4 + 3y} = \frac{x}{1} \quad \text{cross multiply!} \]
7-6y=x(4+3y)=4x+3xy
-4x+6y
7-4x=3xy+6y=y(3x+6)
/3x+6
/3x+6
\[ f^{-1}(x) = \frac{7 - 4x}{3x + 6} \]
\[
\frac{7 - 4f(x)}{3f(x) + 6} = \frac{\frac{7}{1} - \frac{4}{1} \times \frac{7 - 6x}{4 + 3x}}{\frac{3}{1} \times \frac{7 - 6x}{4 + 3x} + \frac{6}{1}} = \frac{\frac{7}{1} - \frac{28 - 24x}{4 + 3x}}{\frac{21 - 18x}{4 + 3x} + \frac{6}{1}}
\]
\[
= \frac{\frac{7(4 + 3x)}{1(4 + 3x)} - \frac{28 - 24x}{4 + 3x}}{\frac{28 + 21x}{1(4 + 3x)} - \frac{28 - 24x}{4 + 3x}}
\]
\[
= \frac{\frac{28 + 21x}{1(4 + 3x)} - \frac{28 - 24x}{4 + 3x}}{1(4 + 3x)}
\]
\[
= \frac{\frac{21 - 18x + 24 + 18x}{4 + 3x}}{\frac{45x}{1(4 + 3x)}}
\]
\[
= \frac{\frac{45x}{1(4 + 3x)} \times \frac{4 + 3x}{45}}{x}
\]
Quiz
Find the exponential function through (3,8) and (9,2)

From <https://mobile.twitter.com/Math4Sher/status/5352574528177308787pzy>

\[ y = cb^x \]
\[ \frac{2}{8} = \frac{b^9}{b^3} \]
\[ 2/8 = 0.25 = b^6 \]
\[ b = 0.25^{(1/6)} = 0.7937 \]
\[ 8 = c0.7937^3 \]
\[ 8/0.7937^3 = 16.0 \]
\[ y = 16 \times 0.7937^x \]
\[ 16 	imes 0.7937^9 = 2.0 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14810.31</td>
</tr>
<tr>
<td>5</td>
<td>16086.41</td>
</tr>
<tr>
<td>8</td>
<td>16321.71</td>
</tr>
<tr>
<td>12</td>
<td>16826.6</td>
</tr>
</tbody>
</table>

\[ \frac{16086.41}{14810.31} = \frac{c b^5}{c b^2} = b^3 \]
\[ (16086.41/14810.31)^{(1/3)} = 1.0279 = b \]
\[ 14810.31 = c 1.0279^2 \]
\[ 14810.31/1.0279^2 = 14017.237 = c \]
\[ 14017.237 \times 1.0279^x \text{ month} \]
\[ 14017.237 \times 1.0279^8 = 16084.8401 \text{ check} \]

\[ \frac{16826.6}{16321.71} = \frac{c b^{12}}{c b^8} = b^4 \]
\[ (16826.6/16321.71)^{(1/4)} = 1.0076 = b \]
\[ 16321.71 = c 1.0076^8 \]
\[ 16321.71/1.0076^8 = 15362.4466 = c \]
\[ 15362.4466 \times 1.0076^x \text{ month} \]
\[ 15362.4466 \times 1.0076^{12} = 16823.5751 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14810.31</td>
</tr>
<tr>
<td>12</td>
<td>16826.6</td>
</tr>
<tr>
<td>12</td>
<td>16826.6</td>
</tr>
</tbody>
</table>

\[ \frac{16826.6}{14810.31} = \frac{c b^{12}}{c b^2} \]
\[ (16826.6/14810.31)^{(1/10)} = 1.0128 = b \]
\[ 14810.31 = c 1.0128^2 \]
\[ 14810.31/1.0128^2 = 14438.3233 = c \]
\[ 14438.3233 \times 1.0128^x \text{ month} \]
14438.3233*1.0128^12 = 16819.035

14017.237*1.0279^17.8 = 22876.4079
15362.4466*1.0076^17.8 = 17578.8157
14438.3233*1.0128^17.8 = 18106.6661

The green (last 6 months) function has the least error and it also has the most accurate prediction for 11/20 so it is the best.
Exponential had the best prediction and the lowest error so it is the best function.
\[(1/8)^{1/12} = 0.8409 \text{ b}\]
\[1/0.8409^9 = 4.7566 \text{ c}\]
\[y = 4.7566 \times 0.8409^x\]
\[4.7566 \times 0.8409^{-3} = 7.9995\]
8. \[ P \left( 1 + \frac{r}{n} \right)^{nt} = F \]

\[ P = 50 \quad r = 0.06 \quad n = 12 \quad t = 3 \]

\[ 50 \times (1 + 0.06/12)^{(12 \times 3)} = 59.83 \]

14. \[ Pe^{rt} \]

\[ P = 400 \quad r = 0.07 \quad t = 3 \]

\[ 400 \times 2.71828^{(0.07 \times 3)} = 493.47 \]

17. \[ F = 1000 \quad r = 0.06 \quad n = 365.24 \quad t = 2.5 \]

\[ P \left( 1 + \frac{0.06}{365.24} \right)^{365.24 \times 2.5} = 1000 \]

\[ 1000 / \left( 1 + 0.06/365.24 \right)^{(365.24 \times 2.5)} = 860.72 \]

What interest rate will cause my $500 to increase to $700 in 5 years compounded quarterly?
$700 in 5 years compounded quarterly

$$500 \left(1 + \frac{r}{4}\right)^{4\times5} = 1000$$

Supply and Demand
We can sell 100 iPads at $600 each but if we charge $400 each we sell 300 iPads.

$$\frac{100}{300} = \frac{cb^{600}}{cb^{400}} \quad \frac{1}{3} = b^{200}$$

$$(1/3)^{(1/200)} = 0.9945 \quad b$$

$$\frac{100}{0.9945^{600}} = 2,736.072 \quad c$$

$$D(p) = 2736.072\times0.9945^p \quad \text{demand function}$$

$$2736.072\times0.9945^{400} = 301.3301 \quad \text{check}$$

We can buy (on eBay) 20 iPads for $150 each but the next cheapest has 80 iPads for $200 each.

<table>
<thead>
<tr>
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<th>Q</th>
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</thead>
<tbody>
<tr>
<td>150</td>
<td>20</td>
<td>20*150=3,000</td>
</tr>
<tr>
<td>+</td>
<td>80</td>
<td>80*200 = 16,000</td>
</tr>
<tr>
<td>19000/100 = 190</td>
<td>20+80=100</td>
<td>3000+16000=19000</td>
</tr>
</tbody>
</table>

$$\frac{100}{20} = \frac{cb^{190}}{cb^{150}} \quad 5 = b^{40}$$
$5^{(1/40)} = 1.0411 \text{ b}$

$20/(1.0411^{150}) = 0.0476 \text{ c}$

$S(p) = 0.0476*1.0411^p \quad \text{supply function}$

$0.0476*1.0411^{190} = 100.2702 \quad \text{check}$

Equilibrium

$2736.072 \times 0.9945^p = 0.0476 \times 1.0411^p$

$/0.0476 \quad /0.0476$

$2736.072 \times 0.9945^p \quad 0.0476$

$0.0476 \quad 0.9945^p \quad /0.9945^p$

$2736.072 \times 1.0411^p \times (0.9945)^p$

$log_{0.0476} \frac{2736.072}{0.9945} = p \log_{0.9945} \frac{1.0411}{0.9945}$

$log \frac{2736.072}{0.0476} = p$

$log \frac{1.0411}{0.9945}$

$Log(2736.072/0.0476)/log(1.0411/0.9945) = 239.32 \text{ equilibrium price}$

$2736.072 \times 0.9945^{239.32} \times 730.9796 \text{ iPads equilibrium quantity}$

$0.0476 \times 1.0411^{239.32} \times 730.9695 \text{ iPads check}$
quiz $5 sells 7 $8 sells 1 exponential demand? cheapest 2 for $3 each next 8 for $5 each supply? equilibrium price&quant?

From <https://mobile.twitter.com/Math4Sher/status/540283357775794176?v>

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>1 = cb^8</th>
<th>1 = b^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
<td>7 = cb^5</td>
<td>7 = b^3</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>(1/7)^((1/3)) = 0.5228 b</td>
<td>7/0.5228^5 = 179.2339 c</td>
</tr>
</tbody>
</table>

D(p) = 179.2339*0.5228^p
179.2339*0.5228^8 = 1.0002 check

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>3*2=6</td>
</tr>
<tr>
<td>+</td>
<td>8</td>
<td>5*8=40</td>
</tr>
<tr>
<td>46/10=4.6</td>
<td>2+8=10</td>
<td>6+40=46</td>
</tr>
</tbody>
</table>

S(p) = 0.0978*2.7344^p
0.0978*2.7344^4.6 = 9.9978  check

Equilibrium

$$179.2339 * 0.5228^p = 0.0978 * 2.7344^p$$

$$/0.0978 \quad 0.5228^p \quad /0.0978 \quad 0.5228^p$$

$$\log_{0.0978} 179.2339 = \frac{2.7344^p}{0.5228^p} = \left(\frac{2.7344}{0.5228}\right)^p$$

$$\log_{0.0978} 179.2339 = p \log_{0.0978} 2.7344$$

$$/\log 2... \quad / \log 2...$$

$$\log_{0.0978} 179.2339 = p \log_{2.7344} 0.5228$$

$$\log_{2.7344} 0.5228$$
Log(179.2339/0.0978)/log(2.7344/0.5228) = 4.54 equilibrium price
179.2339 \times 0.5228^ {4.54} = 9.43 equilibrium quantity
0.0978 \times 2.7344^ {4.54} = 9.41 check

180° = \pi \text{ radians}
30° = \frac{\pi}{6} \text{ radians}
\[0.1, \frac{\pi}{2}, -\frac{3\pi}{2}
\]

\[0.7, 0.7\]

\[-1, \frac{\pi}{2}, -\frac{\pi}{2}\]

\[1.0, -2\pi, 0, 2\pi\]

\[-0.7, -0.7\]

\[0.7, -0.7\]

<table>
<thead>
<tr>
<th>( \cdot )</th>
<th>(-2\pi)</th>
<th>(-\frac{7\pi}{4})</th>
<th>(-\frac{3\pi}{2})</th>
<th>(-\frac{5\pi}{4})</th>
<th>(-\frac{\pi}{2})</th>
<th>(-\frac{\pi}{4})</th>
<th>(0)</th>
<th>(\frac{\pi}{4})</th>
<th>(\frac{\pi}{2})</th>
<th>(\frac{\pi}{4})</th>
<th>(\frac{3\pi}{4})</th>
<th>(\frac{\pi}{2})</th>
<th>(\frac{3\pi}{4})</th>
<th>(\frac{5\pi}{4})</th>
<th>(\frac{3\pi}{2})</th>
<th>(\frac{7\pi}{4})</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\cos)</td>
<td>1</td>
<td>0.7</td>
<td>0</td>
<td>-1</td>
<td>-0.7</td>
<td>0</td>
<td>0.7</td>
<td>1</td>
<td>0.7</td>
<td>0</td>
<td>-1</td>
<td>-0.7</td>
<td>0</td>
<td>0.7</td>
<td>1</td>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td>(\sin)</td>
<td>0</td>
<td>0.7</td>
<td>1</td>
<td>0</td>
<td>-0.7</td>
<td>0</td>
<td>0.7</td>
<td>1</td>
<td>0.7</td>
<td>0</td>
<td>-0.7</td>
<td>0</td>
<td>0.7</td>
<td>1</td>
<td>0.7</td>
<td>0</td>
<td>-0.7</td>
</tr>
</tbody>
</table>
The size of the circle is the amplitude
Altitude is the middle value of the curve
\( altitude + amplitude \times \sin(x) \)
\( 5 + 2\sin(x) \)
WINDOW
Xmin = -6.3
Xmax = 6.3
Xscl = \pi/4
Ymin = -10
Ymax = 10
Yscl = 1

\sin
\cos

2-5\cos(x)
$3 - 3\sin(x)$

Altitude + amplitude (cos or sin) (frequency * x)
quiz Graph $-6\sin(x)$ from $-2\pi$ to $2\pi$ by $1/4$ pi's

Altitude + amplitude (cos or sin)(frequency * x)

2 + 4 $\sin(\frac{2}{2}x)$

Period = length of a cycle

Period * frequency = $2\pi$  \hspace{1cm} \frac{2p}{2} = 2\pi

Altitude = 2
Amplitude = 4
Frequency = 2
Period = $\pi$
-7 + 2\cos(4x)

4\pi = 2\pi \quad p = \frac{\pi}{2}
Quiz
Graph $3 + 5\sin(2x)$ from $-2\pi$ to $2\pi$ by $1/4 \pi$'s

Frequency = 2
$f \cdot p = 2\pi$
$2p = 2\pi$
$p = \pi$

$8 - 2\cos(2x - 3\pi/2)$
Frequency = 2
Period = $\pi$
$2x - 3\pi/2 = 0$
$2x = 3\pi/2$
x = $3\pi/4$ phase

$1 + 3\cos(x/2)$
Frequency = $1/2$
$1/2p = 2\pi$
p = $4\pi$

$-7 - 2\sin(x + \pi/4)$
-7 - 2\sin(x + \pi/4)

\begin{align*}
x + \frac{\pi}{4} &= 0 \\
-x - \frac{\pi}{4} &= 0 \\
x &= -\frac{\pi}{4}
\end{align*}

Phase shift
1. **30 pts**: Graph the function \( 7 - 3 \cos \left( \frac{x}{2} + \frac{3\pi}{8} \right) \) from \(-2\pi\) to \(2\pi\).

\[ \frac{x}{2} + \frac{3\pi}{8} = 0 \]

\[ \frac{x}{2} = -\frac{3\pi}{8} \]

\[ x = -\frac{3\pi}{4} \]

Phase: \(-\frac{3\pi}{4}\)

2. **30 pts**: Graphically and algebraically find the intersection of \(-3x^2 + 30x - 68\) and the line through \(5,5\) and \(7,7\).

\[ x = \frac{3\pi}{8} \]

Period: \(4\pi\)

3. **40 pts**: A bookstore sells 400 books of graph paper at $7.00 a book but if they charge $9.00 a book they sell 100 books. They have a supplier for 100 books at $3.00 a book but the next cheapest supplier will sell 200 books at $3.50 a book.

a. Find an exponential demand function.

b. Find an exponential supply function.

c. Find the equilibrium price and demand for the exponential functions.
Algebraically
Slope = \frac{(-7-5)}{(7-5)} = -6
Intercept = -7 - 6*7 = 35
y = -6x + 35
-6*5 + 35 = 5 check
-6x + 35 = -3x^2 + 30x - 68
+6x - 35
0 = -3x^2 + 36x - 103
Helper h = 36/(-2*3) = 6
Helper k = -3*6^2 + 36*6 - 103 = 5

x values = 6 ± \sqrt{\frac{5}{-3}}
6+sqrt(-5/-3) = 7.29
-6*7.29 + 35 = -8.74 \quad (7.29,-8.74)
-3*7.29^2 + 30*7.29 - 68 = -8.7323 check
6-sqrt(-5/-3) = 4.71
-6*4.71 + 35 = 6.74 \quad (4.71,6.74)
-3*4.71^2 + 30*4.71 - 68 = 6.7477 check
3. 

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
400 &= cb^7 \\
600 &= cb^5 \\
\frac{2}{3} &= b^2 \\
(2/3)^{(1/2)} &= 0.8165 = b \\
600/0.8165^5 &= 1,653.371 = c \\
D(p) &= 1653.371*0.8165^p \\
1653.371*0.8165^7 &= 400.0034 \text{ check}
\end{align*}
\]

<table>
<thead>
<tr>
<th>P</th>
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</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>100</td>
<td>3*100=300</td>
</tr>
<tr>
<td>+200</td>
<td>3.5*200=700</td>
<td></td>
</tr>
<tr>
<td>1000/300=10/3=3 1/3</td>
<td>100+200=300</td>
<td>300+700=1,000</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
300 &= cb^{1\frac{1}{3}} \\
100 &= cb^3 \\
\frac{3}{2} &= b^2 \\
27 &= b \\
100/27^3 &= 0.0051 = c \\
S(p) &= 0.0051*27^p \\
0.0051*27^{(10/3)} &= 301.1499 \text{ check}
\end{align*}
\]

\[
\begin{align*}
1653.371 * 0.8165^p &= 0.0051 * 27^p \\
/0.0051*0.8165^p &/0.0051*0.8165^p
\end{align*}
\]

\[
\begin{align*}
1653.371 &= \left(\frac{27}{0.8165}\right)^p \\
\log \frac{0.0051}{1653.371} &= p \log \frac{27}{0.8165} \\
/\log(27/0.8165) &/\log(27/0.8165)
\end{align*}
\]

\[
\begin{align*}
\log(1653.371/0.0051)/\log(27/0.8165) &= 3.63 \text{ equilibrium price} \\
1653.371*0.8165^{3.63} &= 792 \text{ equilibrium demand} \\
0.0051*27^{3.63} &= 800.6093 \text{ check}
\end{align*}
\]