2003 Final Exam
Answers, Part I

1. D
2. Thrust is multiplied by 16.
3. B
4. $P = 14,780 + 780t$
5. $y = 2x + 4$
6. $\{x \mid x \neq -3, x \neq 3\}$
7. $x = 28$
8. $9x^2 + 6x - 8$
9. $f^{-1}(x) = \frac{2\sin x}{3}$
10. 1
11. C
12. E
13. $g(x) = f(-x) + 1$
14. 1
15. $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$
16. C
17. $\frac{1}{6}$
18. $y =$
19. B
20. 100
21 B
22. $P(x) = \frac{1}{15}(x+5)(x+2)(x-6$
23. D
24. $y = -2\cos\left(\frac{1}{5}x\right) + 2$
25. D

2003 Final Exam
Solutions, Part II

1. a. $15C + 5t = 150 \rightarrow 15C = 150 - 5t \rightarrow C = 10 - \frac{1}{3}t.$

   b. The horizontal intercept, 30, represents the number of cassette tapes Juan can buy if he buys no CDs. The vertical intercept, 10, represents the number of CD's Juan can buy if he buys no cassette tapes.

   c. The slope, $-\frac{1}{3}$, tells us that for each 3 tapes Juan buys, he can buy 1 fewer CD. Alternatively, for each 1 less CD Juan buys, he can buy 3 more cassette tapes.

   d. If the price of CDs doubled, the slope would be $-\frac{1}{6}$. The horizontal intercept of the graph remains the same, 30. The vertical intercept of the graph would be halved to 5.

2. a. False b. False c. True d. False e. False

3. a. The C intercept tells us the temperature of the cake when it was initially removed from the freezer.

   b. After a long period of time (as t gets large, $e^{-0.153t}$ approaches 0. The temperature of the cake, C, approaches $70(1.05) = 73.5°F$. This corresponds to a horizontal asymptote on the graph of this function.

   c. To find the time when the cake reaches 20°:

$$20 = 70(1.05 - e^{-0.153t}) \rightarrow \frac{20}{70} = 1.05 - e^{-0.153t} \rightarrow e^{-0.153t} = 1.05 - \frac{20}{70}$$

$$\ln(e^{-0.153t}) = \ln(1.05 - \frac{20}{70})$$

$$-0.153t = \ln(1.05 - \frac{20}{70}). \text{ Therefore,}$$

$$t = \frac{\ln(1.05 - \frac{20}{70})}{-0.153} \text{ or approximately 2 minutes.}$$

   To find the time when the cake reaches 30°:

$$30 = 70(1.05 - e^{-0.153t}) \rightarrow \frac{30}{70} = 1.05 - e^{-0.153t} \rightarrow e^{-0.153t} = 1.05 - \frac{30}{70}$$

$$\ln(e^{-0.153t}) = \ln(1.05 - \frac{30}{70})$$

$$-0.153t = \ln(1.05 - \frac{30}{70})$$

$$t = \frac{\ln(1.05 - \frac{30}{70})}{-0.153}, \text{ or about 3 minutes.}$$

The soonest you can cut the cake is 2 minutes after is comes out of the freezer. The latest you can cut the cake is 3 minutes after it comes out of the freezer.

4. a. $C(p) = 1.5 + \dfrac{80000}{p}$.

b. In the long run as p gets large, the term $\dfrac{80000}{p}$ approaches zero and $C(p)$ behaves like the function $C(p) = 1.5$. Therefore, the average cost per pint approaches \$1.50 as p gets large. The economic significance is that the initial cost \$80,000 has less of an effect on the average cost per pint as the company produces increasingly more pints of ice cream.

c. $C = 1.5 + \dfrac{80000}{p}$. Solving for p we have:

$$\begin{aligned} Cp &= 1.5p + 80000 \\ Cp - 1.5p &= 80000 \\ p(C - 1.5) &= 80000 \\ p &= \frac{80000}{C - 1.5} \\ f^{-1}(C) &= \frac{80000}{C - 1.5}. \end{aligned}$$

The inverse function, C^{-1}, takes as input an average cost per pint of ice cream, *C*, and outputs the number of pints of ice cream, *p*, that produces that given average cost.

d. $f^{-1}(C) = \dfrac{80000}{C - 1.5} \rightarrow f^{-1}(3) = \dfrac{80000}{3 - 1.5} = \dfrac{80000}{1.5} \approx 53,333.33.$

The company's average cost is \$3.00 per pint if it produces 53,333.33 pints of ice cream. To make a profit, 53,334 pints of ice cream should be produced to achieve an average cost less than \$3.00 per pint.

5. a.

x	-4	-2	0	2	4
$f(x)$	3	5	7	-2	-8
$-f(-x) + 7$	15	9	0	2	4
$f\left(\frac{1}{2}x - 1\right)$	?	5	?	7	?
$2f(x) + 1$	7	11	15	-3	-15

5. b.

x	-3	-2	-1	0	1	2	3
$f(x)$	9	5	3	0	-3	-5	-9
$-f(-x) + 7$	-9	-5	-3	-1	-3	-5	-9

6. a. $h(t) = 15\cos\left(\frac{\pi}{4}t\right) + 18$

   $h(4) = 15\cos(\pi) + 18 = 15(-1) + 18 = 3.$

   You are 3 meters above the ground at time $t = 4$.

   b. The time required for one revolution is the period of the function. Period $= \frac{2\pi}{\frac{\pi}{4}} = 8$ minutes.

   c. The radius of the wheel is the amplitude of the function. Radius = 15 meters.

   d. $30 = 15\cos\left(\frac{\pi}{4}t\right) + 18$

   $\frac{12}{15} = \cos\left(\frac{\pi}{4}t\right)$

   $\frac{\pi}{4}t = \cos^{-1}\left(\frac{12}{15}\right)$

   $t = \frac{\cos^{-1}\left(\frac{12}{15}\right)}{\frac{\pi}{4}} \approx 0.82$ minutes.

   A person is 30 meters or more above the ground from $t = 0$ to $t = 0.82$ minutes. Using the symmetry of the curve, a person is 30 meters or more above the ground for 1.64 minutes.

7. a. Amplitude, $A = \frac{\text{maximum} - \text{minimum}}{2}$

   $$\text{A} = \frac{5500 - 4500}{2} = 500.$$

   $\text{Period} = 1 \text{ year} \rightarrow \text{B} = \frac{2\pi}{\text{period}} = \frac{2\pi}{1} = 2\pi.$

   midline = maximum – amplitude = 5000.

   Since the function starts at its minimum, reaches its maximum halfway through its cycle, and then returns back to its minimum, it is modeled by the function $y = -\cos x$.
   Therefore, $P_A(t) = -500\cos(2\pi t) + 5000$.

   b. $P_B(t) = 4000(1.08)^t$

   c. $P_C(t) = 8000 - 250t$

d. $P(t) = 12{,}530e^{-0.085t} = 10{,}000$

$$e^{-0.085t} = \frac{10{,}000}{12{,}530}$$

$$-0.085t = \ln\left(\frac{10{,}000}{12{,}530}\right)$$

$$t = \frac{\ln\left(\frac{10{,}000}{12{,}530}\right)}{-0.085}$$

$$t \approx 2.65 \text{ years.}$$

After about 2.65 years, the population reaches 10,000 people.

8. a. $g(0) = c$ and $g(c) = b$

points: $(0, c)$ and (c, b)

slope: $\frac{b-c}{c-0} = \frac{b-c}{c}$

$g(x) = c + \frac{b-c}{c}x$

b. $g(a) = a$

$f(a) = a$

$f(g(a)) = a$

c. $f(c) = c$

$g(c) = b$

$g(f(c)) = b$

e. $[0, a]$