NCC Precalculus Partnership Program
Final Examination, 2003

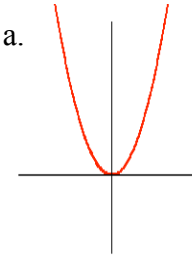
Part I: Answer all 25 questions. Place your answers on the answer sheet provided. No partial credit will be given.
(50 points)

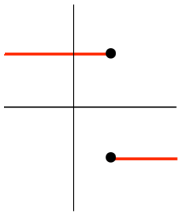
1. If $Q = f(t)$, which of the following represents the average rate of change of Q from $t = a$ to $t = b$?

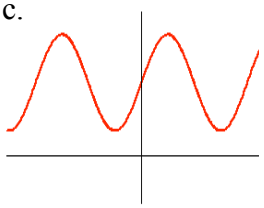
   a. $\dfrac{f(\Delta Q)}{\Delta t}$ b. $\dfrac{b-a}{f(b)-f(a)}$ c. $\dfrac{b-a}{\Delta t}$ d. $\dfrac{f(b)-f(a)}{b-a}$ e. none of these

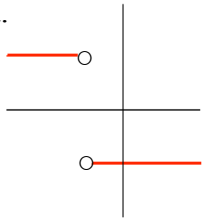
2. The thrust, T, in pounds, delivered by a ship's propeller is directly proportional to the square of the propeller's speed, r, in rotations per minute, times the fourth power of the propeller's diameter, d, in feet. What happens to the thrust if the propeller diameter, d, is doubled?

3. Which of the following graphs does not represent a function?

   a. 

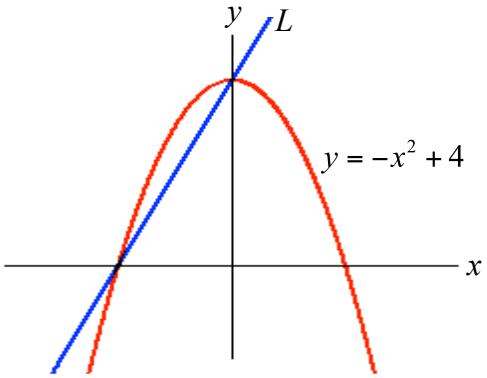
   b. 

   c. 

   d. 

4. In 1950, the population of a town was 14,780, and has been growing by a constant rate of 780 people per year. Find a formula for P, the town's population, in terms of t, the number of years since 1950.

5. Find an equation of the line L, shown in the figure below.



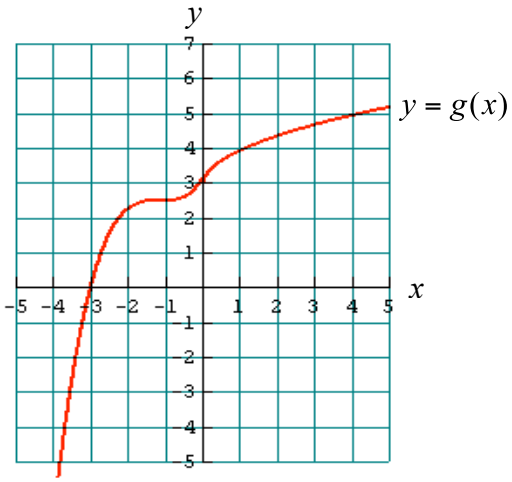
6. What is the domain of $y = \dfrac{x}{x^2 - 9}$?

7. Solve for x: $\log_2(x+4) - 4 = 1$

8. If $g(x) = x^2 + 4x - 7$, express $y = g(3x-1) + 2$ in simplest form.

9. Find a *formula* in terms of x for $f^{-1}(x)$ if $f(x) = \arcsin\left(\frac{3x}{2}\right)$.

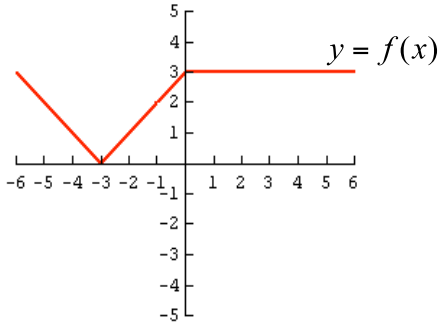
10. The graph of $g(x)$ is shown below. Determine $g^{-1}(4)$.



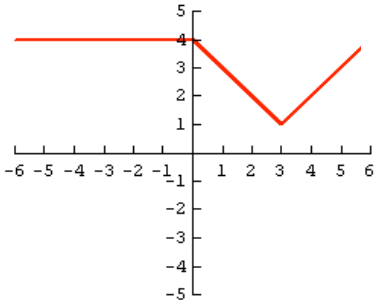
11. Of the following sets of ordered pairs, which represents an invertible function?
    a. {(6,1)} b. {(2,1), (3,1), (4,1), (5,1)} c. {(2,1), (3,6), (4,7), (2,8)} d. {(5,3), (5,2), (5,1)}

12. Microbe population A grows at 5,000 cells per hour. Microbe population B grows at 0.02% per hour. Assuming population A initially has $2P_o$ cells and population B initially has P_o cells, then,
    a. Population A will always be larger than population B
    b. Population B will never be larger than population A
    c. Population A will always be twice as large as population B
    d. Population A and population B will eventually become equal and remain equal to each other
    e. None of these

13. The graph of $y = f(x)$ is shown to the right.



Write a formula for the graph of $g(x)$ terms



14. Let $N(t) = 30 + 20\cos\left(2\pi\left(t - \frac{1}{4}\right)\right)$. What is the period of the graph of $N(t)$?

15. What is the *exact* range of the function: $f(x) = \tan^{-1}[\sin(x)]$?

16. If $f(x) = e^x$ and $g(x) = \ln(2x+1)$, for $x > \frac{-1}{2}$, which of the following represents $f(g(x))$?

    a. $\ln(2e^x + 1)$ b. e^{2x+1} c. $2x+1$ d. $2e^x + 1$ e. none of these

17. If a function f is invertible, then $f^{-1}(f(\frac{1}{6}))$ is

    a. 6 b. $\frac{1}{6}$ c. 0 d. $-\frac{1}{6}$ e. cannot be determined

18. What are the horizontal asymptote(s) for the graph of the function $h(x) = \dfrac{x+150}{x^2 - 25}$?

19. As $x \to \infty$ which of the following functions has the largest y values?

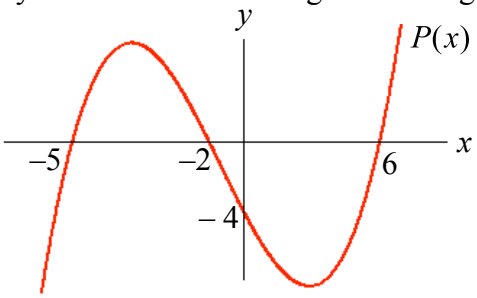
    a. $y = 4x^{\frac{1}{3}} + 78x$ b. $y = x^{\frac{5}{4}} + 1$ c. $y = x^{0.47} + 10^8$ d. $y = -x^2 + x$ e. $y = e^{-x} + 1000$

20. At how many points do $f(x) = \cos(x)$ and $g(x) = \sin(x)$ intersect on the interval $0 \le x \le 100\pi$?

21. The price of a certain item increases due to inflation. Let $p = f(t) = 7.50(1.058)^t$ be the price of the item as a function of the time in years, with t = 0 in 1970. Therefore, $f^{-1}(20)$ tells us which of the following?

    a. The price of the item in 1990.
    b. The number of years it takes for the price to become $20.00.
    c. The price of twenty items in t years.
    d. In twenty years the price of the $7.50 item will be $20.00 more than in 1970.
    e. None of the above.

22. Write an equation of the polynomial of smallest degree whose graph is shown below.



23. What happens to $f(x) = \dfrac{10 - 155x + 16x^2 - 4x^3}{5x^3 + 342x^2 - 7{,}932}$ as $x \to +\infty$?

    a. $f(x) \to -4$

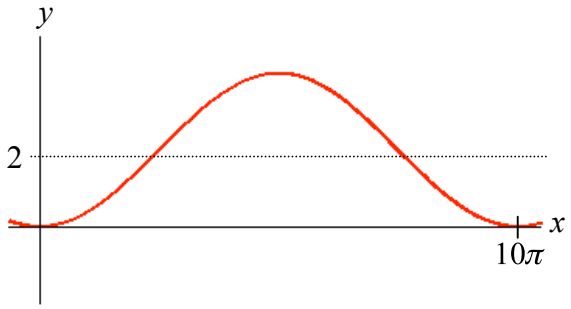
    b. $f(x) \to -7{,}932$

    c. $f(x) \to \dfrac{4}{7{,}932}$

    d. $f(x) \to -\dfrac{4}{5}$

    e. none of these

24. Find a possible formula for the trigonometric function whose graph is shown below.



25. If $f(t) = 1.75(0.85)^t$ represents the number of people (in thousands) present at a rally at time t, measured in hours, then

    a. the number of people at the rally is increasing 85% per hour.

    b. the number of people at the rally is increasing 15% per hour.

    c. the number of people at the rally is decreasing 85% per hour

    d. the number of people at the rally is decreasing 15% per hour.

    e. none of these

Part II: Answer only **5** questions on this part. Be sure you clearly indicate the questions you do not wish to be graded. Show all work. **(50 points)**

1. Juan received $150.00 for his birthday. He decided to spend it all on CDs and cassette tapes. Each CD costs $15.00 and each cassette tape costs $5.00. **(3,2,2,3)**

    a. Write a formula for the number of CDs he can buy, C, as a function of the number of cassette tapes, t.

    b. If $C(t)$ was graphed, what would the horizontal and vertical intercepts represent in the context of this problem?

    c. If $C(t)$ was graphed, what would the slope represent in the context of this problem.

    d. Suppose the price of CDs suddenly doubled. How does this change affect the slope, and the horizontal and vertical intercepts of the graph of the function? *Be specific.*

2. Answer true or false. **(10)**
   a. $y = f(-x)$ is a reflection of $y = f(x)$ about the x-axis.
   b. In general, $f(4) + g(4) = (f + g)(8)$.
   c. If $y = A\sin B(x - h) + k$ then its graph is the graph of $y = A\sin(Bx) + k$ shifted h units to the right.
   d. If a population doubles in size every 20 years, then its annual continuous growth rate is approximately 1.5%.
   e. If a and b are positive numbers, then the $\ln(a + b) = \ln a + \ln b$.

3. The temperature, in degrees Fahrenheit, of an ice cream cake t minutes after it is removed from the freezer is given by $C(t) = 70(1.05 - e^{-0.153t})$, where t is measured in minutes since the cake was removed from the freezer. **(2,3,5)**
   a. What does the C intercept tell you about the cake?
   b. What will happen to the temperature of the cake after a long period of time? To what feature of the graph of this function does this correspond?
   c. If the cake becomes too warm, it will melt. If it is too cold, I cannot cut it. I can cut the cake if its temperature is greater than 20°. However, if its temperature rises above 30° it will melt before I can eat it. When is the soonest and latest I can cut the cake? Express your answer both *exactly* and then rounded to the nearest minute

4. Suppose that it costs a company \$80,000 to begin producing pints of ice cream, plus \$1.50 for each pint produced. Let p be the number of pints produced by the company. **(3,2,3,2)**
   a. The company's *average* cost *per* pint $C(p)$, is defined to be the total cost divided by the number of pints, p, produced. Find a formula for $C(p)$.
   b Explain the economic significance of the ''long range'' behavior of $C(p)$.
   c. Find a formula for C^{-1} and explain its economic significance.
   d. The company makes a profit if its average cost is less than \$3.00. Find the minimum number of units the company must produce to make a profit.

5. a. Recopy the table below in you bluebook. Fill in the blanks of the table for which you have sufficient information. Put a question mark (?) in any box for which there in not enough information to obtain an answer. **(2 points per row)**

x	-4	-2	0	2	4
$f(x)$	3	5	7	-2	-8
$-f(-x)+7$					
$f(\frac{1}{2}x-1)$					
$2f(x)+1$					

   b. Recopy the table below in you bluebook. If $g(x)$ is an odd function and $h(x)$ is an even function, fill in the blanks of the table for which you have sufficient information. Put a question mark (?) in any box for which there in not enough information to obtain an answer. **(2 points per row)**

x	-3	-2	-1	0	1	2	3
$g(x)$	9	5	3	0			
$h(x)$	-9	-5	-3	-1			

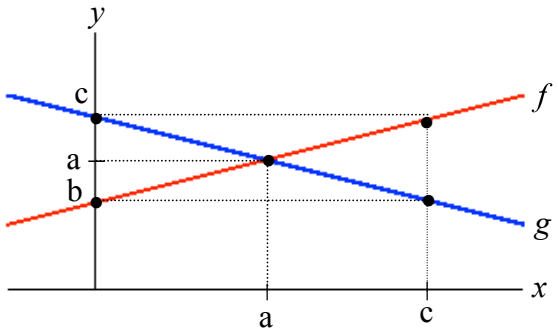
6. Suppose you are on a Ferris wheel and that your height, in meters, above the ground at time t, in minutes is given by $h(t) = 15\cos\left(\frac{\pi}{4}t\right) + 18$. **(2,2,2,4)**

   a. How high above the ground are you at time $t = 4$?
   b. How long does one revolution take?
   c. What is the radius of the wheel?
   d. During one revolution, how much time does one spend 30 meters or more above the ground?

7. City A is a modest tourist town, which means that its population undergoes a seasonal variation. On January 1, 1990, the population dips down to 4,500 people, but on July 1st, with the warm weather, its population climbs to around 5,500 people. By the following January 1st, however, the population has again fallen to 4,500 people. This trend repeats every year. City B, on the other hand, is a small town not far from City A. Its population has been growing extremely rapidly ever since the arrival of several large, new automobile plants. There were only 4,000 people there on January 1, 1990, but its population has grown by 8% every year thereafter. A third neighboring town, City C, had a population of 8000 people on January 1, 1990, but is on the verge of becoming a "ghost town". Since a large clothing manufacturer moved its headquarters, the town's population has been decreasing by 250 people per year. (*Assume that* $t = 0$ *means January 1, 1990*) **(3,2,2,3)**

   a. Find a possible formula for $P_A(t)$, the population of city A in year t.
   b. Find a formula for $P_B(t)$, the population of city B in year t.
   c. Find a formula for $P_C(t)$, the population of city C in year t.
   d. Suppose the population of a town is modeled by the function $P(t) = 12{,}530e^{-0.085t}$, where t is measured in years. Find an *exact* and approximate value for t, the time at which the population reaches 10,000 people?

8. The graphs of f and g are given below. **(3,2,2,2,1)**



   a. Assuming g is linear, find its equation.
   b. Evaluate $f(g(a))$.
   c. Evaluate $g(f(c))$.
   d. Evaluate $f^{-1}(b) - g^{-1}(b)$
   e. For what positive value(s) of x is $g(x) \geq f(x)$?