2004 Final Exam
Answers for Part I

1. -2

2. $V = \frac{5}{P}$

3. D

4. People per year

5. \$225.00

6. $y = \frac{7}{3}x - 2$

7. $x < -1$ or $x > 1$

8. $1 + 10^{-2}$ or equivalent

9. 54.9 years

10. A

11. $y = 3(0.5)^x$

12. 4

13. B

14. $y = -\frac{1}{5}(x+3)^2 + 5$ or equivalent

15. C

16. B

17. $e^{-\frac{\pi}{2}} < y < e^{\frac{\pi}{2}}$

18. $f^{-1}(x) = \frac{e^x + 1}{3}$

19. A

20. $h(x) = -f(x+2) + 2$ or
$h(x) = 2 - f(-x)$

21. $x = 0,\ x = -2,\ x = 2$

22. C

23. $y = -\frac{1}{5}(x+5)(x+1)(x-2)$

24. C

25. $y = 3\cos\left(\frac{\pi}{4}x\right) + 3$

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Solutions for Part II

1. a.

x	-4	-2	0	2	4
$f(x)$	2	-2	7	1	-5
$-f(-x)-6$	-1	-7	-13	-4	-8
$f(2-\frac{1}{2}x)$	-5	?	1	?	7
$2-\frac{1}{2}f(x)$	1	3	-1.5	1.5	4.5

b.

x	-3	-2	-1	0	1	2	3
$g(x)$	-6	4	3	0	-3	-4	6
$h(x)$	-6	-1	2	1	2	-1	-6

2. a. The new function can be written as $y = 7 + 3\cos[6(x+3)]$.

b. 1. The amplitude of the function is 200, so this is the radius of the circle. Therefore, the circumference of the circle is $C = 400\pi \approx 1,257$ feet.

2. The time it takes to go once around the track corresponds to the period of the function. The period is expressed as $\frac{2\pi}{B}$, so $P = \frac{2\pi}{\frac{2}{3}} = 3\pi$ sec.

3. $\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{400\pi}{3\pi} \approx 133\,\frac{\text{ft}}{\text{sec}}$.

c. Using a graphing utility or a solver function, the gum is above 9 inches between $t \approx 0.1$ sec. and $t \approx 0.4$ sec.

d. We need to solve $9 = 13\sin(4\pi t - \frac{\pi}{2}) + 13$ for t. This yields $t_1 = \dfrac{\arcsin\left(\frac{-4}{13}\right) + \frac{\pi}{2}}{4\pi}$. From part (c) we know there is a second value, and it is in the 4th quadrant. Realizing that the frequency is 4π, the period is 0.5. So $t_2 = 0.5 - \dfrac{\arcsin\left(\frac{-4}{13}\right) + \frac{\pi}{2}}{4\pi}$. The gum is above 9" between t_1 and t_2.

3. a. F b. F c. T d. T e. F f. T g. F h. F
i. T j. T k. F l. F m. T n. T o. F

4. a. The population is $P(t) = 1,200,000e^{0.0238t}$.

b. The number of people that can be fed is $N(t) = 2,000,000 + 15,000t$.

c. Evaluating $P(8)$ we obtain $P(8) = 1,451,680$. This is the size of the population in 2004.

d. The average rate of change is $\frac{\Delta P}{\Delta t} = \frac{P(8)-P(0)}{8-0} = \frac{1,451,681 - 1,200,000}{8} \approx 31,460.$ people per year.

e. We solve $1,200,000e^{0.0238t} = 2,000,000 + 15,000t \rightarrow t \approx 30$. Therefore in the year 2026, the country's food supply is first unable to feed its population.

f. We solve $1,200,00e^{0.0238t} = 2,000,000 + 30,000t \rightarrow t \approx 42$. The time will not double.

5. a. The points $(3,490)$ and $(5,850)$ are on the supply curve. So, $m = \dfrac{\Delta q}{\Delta p} = \dfrac{850-490}{5-3} = 180.$

   Thus, $q - 490 = 180(p-3)$, or $q_s = 180p - 50.$

   b. The points $(1,180)$ and $(1.50,170)$ are on the demand curve. So,

   $m = \dfrac{\Delta q}{\Delta p} = \dfrac{180-170}{1-1.50} = -20.$ Thus, $q - 180 = -20(p-1)$, or, $q_d = -20p + 200.$

   c. Setting $q_s = q_d$, $180p - 50 = -20p + 200 \rightarrow p = 1.25$. The equilibrium quantity is therefore 175.

   d. The new demand curve will be $q_{d_{new}} = -20(p+.25) + 200$. The supply curve remains unchanged. Therefore, $180p - 50 = -20p + 195 \rightarrow p \approx 1.23$. The equilibrium quantity is $q \approx 171$.

6. a. The equation is $Q = 80e^{-0.17t}$.

   b. The amount of caffeine is immaterial. $t = \dfrac{\ln 0.5}{-0.17} \approx 4$ hours.

   c. The amount of caffeine in the body at 4 PM is $Q = 80e^{-0.17(8)} \Rightarrow Q \approx 20.53$ mg. Therefore, the percent of caffeine in the body at 4 PM is $\dfrac{20.53}{80} \times 100 \approx 26\%$.

   d. The total caffeine is $Q_T = Q_1 + Q_2 = 80e^{-0.17(2)} + 80e^{-0.17(1)} \approx 124$ mg. Alternative algebraic solutions are acceptable.

7. a. The total cost is $C(x) = 30,000 + 3x$.

   b. The average cost is $A(x) = \dfrac{30,000 + 3x}{x}$.

   c. The asymptote is $y = 3$. In economic terms, the start-up cost becomes less and less of a factor as the number of units increases. Therefore, the asymptote ($y = 3$) for the average cost function is the unit cost value.

   d. If $y = \dfrac{30000 + 3x}{x}$, then $x = \dfrac{30000 + 3y}{y}$. Therefore $y = A^{-1}(x) = \dfrac{30000}{x-3}$. This formula allows one to input the average cost. The output will be the number of skateboards that needs to be produced to achieve that average cost.

   e. We want $A(x) = \dfrac{30,000 + 3x}{x} < 5 \rightarrow x > 15,000.$