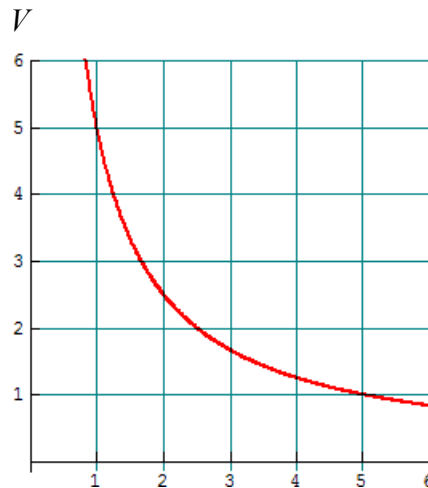


**NCC Precalculus Partnership Program  
Final Examination, 2004**

**Part I:** Answer only 20 of the 25 questions below. Each question is worth 2 points. Place your answers on the answer sheet provided. Write the word **OMIT** next to those questions you wish to exclude. Leave all answers in EXACT form, i.e., in terms of  $e$ ,  $\pi$ ,  $\ln$ ,  $\sqrt{\quad}$ , etc., unless otherwise instructed. No Partial credit will be given.

- Determine the average rate of change of  $f(x) = 4 - \frac{1}{2}(x-3)^3$  with respect to  $x$  between  $x = 0$  and  $x = 4$ .
- When temperature is held constant, the volume,  $V$ , of a quantity of gas is inversely proportional to the temperature,  $P$ . The graph below shows this relationship for a particular temperature.



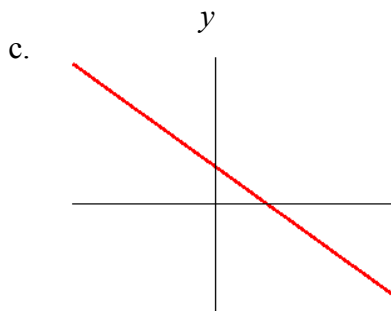
Express  $V$  as a function of  $P$ ?

- Which of the following *does not* represent  $y$  as a function of  $x$ ?

a.  $\{(3,9), (6,9), (9,9)\}$

b. 

$y$	1	2	1	2
$x$	1	2	3	4

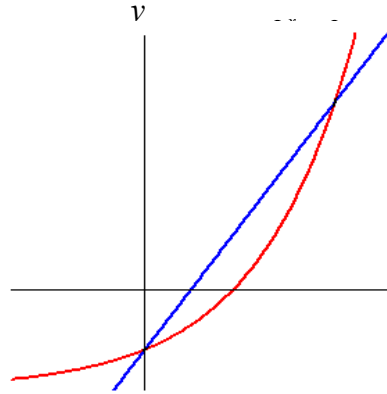


d.  $x$  = the number of days in a month  
 $y$  = the month of the year

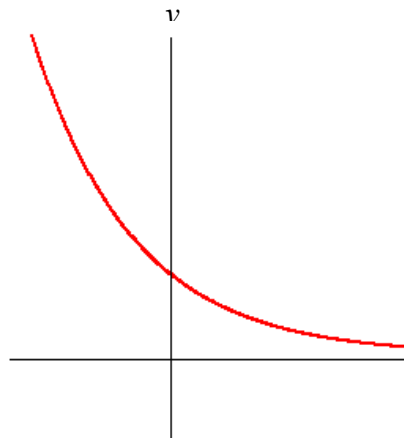
e. none of these

- If  $P(t) = 10,000 - 600t$  represents the population of a town in  $t$  years, what are the units of the slope of the graph of  $P(t)$ ?

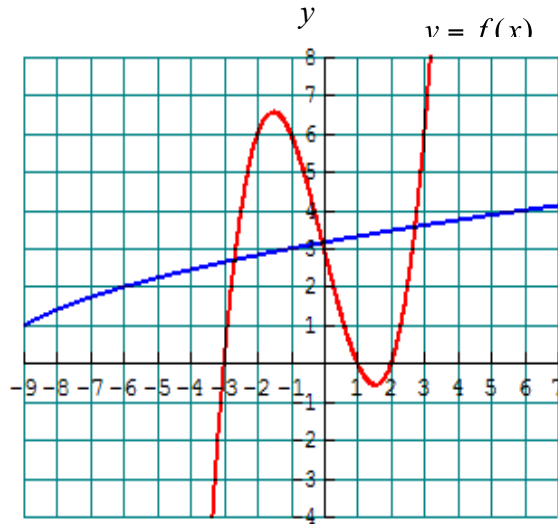
5. I bought a rare book some time ago. The value of the book increased linearly. It was worth \$600 after five years. After eight years, the book was worth \$825. What was the book worth when I bought it?
6. The graphs of line  $L$  and  $y = 2^x - 3$  are shown below. Find the equation of line  $L$ .



7. What is the domain of the function  $f(x) = \frac{x}{\sqrt{x^2 - 1}}$ ?
8. Solve for  $x$ :  $\log(x - 1) + 1 = -1$
9. How many years, to the *nearest tenth of a year*, will it take for an investment to triple if it earns 2% annual interest compounded continuously?
10. If  $g(x) = x^2 - x$ , then  $\frac{g(x+h) - g(x)}{h}$  is equal to
- a.  $2x + h - 1$     b.  $\frac{-2x + h}{h}$     c.  $\frac{h^2 - x}{h}$     d.  $\frac{x^2 + h^2 - x - h}{h}$     e. none of these
11. Write a formula for the exponential function whose graph is shown below.

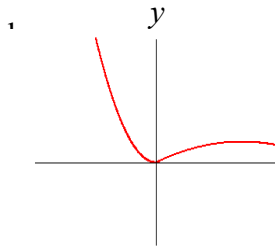


12. The graphs of  $f(x)$  and  $g(x)$  are shown. Find  $g(f(-1))$ .



13. Which of the following *does not* represent an invertible function?

a.  $\{(0, 0)\}$



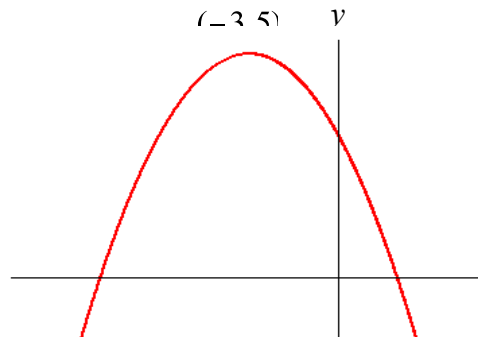
c. The height of a ball dropped from a building,  $h$ , as a function of time,  $t$ .

d. 

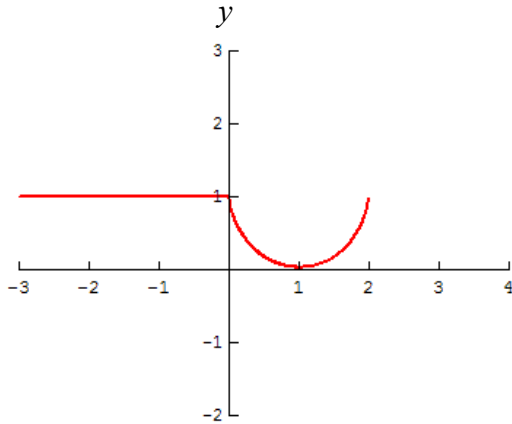
$x$	1	2	3	4
$y$	1	2	3	4

e. none of these

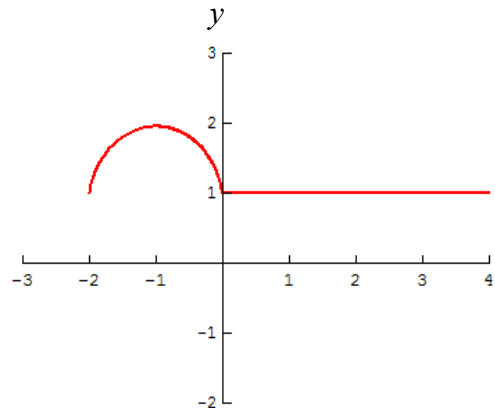
14. Find a formula for the quadratic function whose graph is shown below. (Note that the graph is symmetric about the line  $x = -3$ .)



15. Suppose  $C(x)$  is the cost, in dollars, to buy enough paper goods for a party of  $x$  people. Which of the following is the cost for paper goods if 10 more people come to the party?  
 a.  $C(x)+10$     b.  $C(10x)$     c.  $C(x+10)$     d.  $10C(x)$     e. none of these
16. If a function  $f$  is invertible and if  $-2$  is in the domain of  $f^{-1}$ , then  $f[f^{-1}(-2)]$  is  
 a. 2    b.  $-2$     c.  $\frac{1}{2}$     d.  $-\frac{1}{2}$     e. can't be determined
17. What is the *exact* range of the function  $f(x) = e^{\arctan x}$ ?
18. If  $f(x) = \ln(3x - 1)$ , find a formula in terms of  $x$  for  $f^{-1}(x)$ .
19. As  $x \rightarrow \infty$  which of the following functions has the largest  $y$  value?  
 a.  $y = 4x^{-10} + 1000x + 1$     b.  $y = \sqrt{x} + 10^{23}$     c.  $y = x^{0.67} - 1$   
 d.  $y = -x^2 + 1000$     e.  $y = 2^{-x} + 10^9$
20. The graph of  $y = f(x)$  is shown below.



Write a formula for the graph of  $h(x)$ , shown below, in terms of  $f(x)$ .



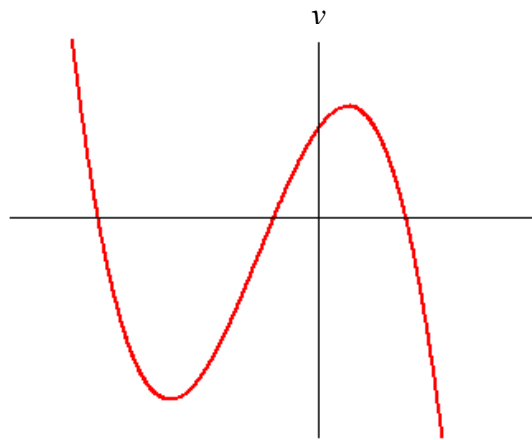
21. Find the equation(s) for the vertical asymptote(s) to the graph of the function

$$h(x) = \frac{1-x}{x^3-4x}.$$

22. Let  $C(t)$  represent the total number of cars sold, in millions,  $t$  years since 1990.

Then  $C^{-1}(10)$  tells us

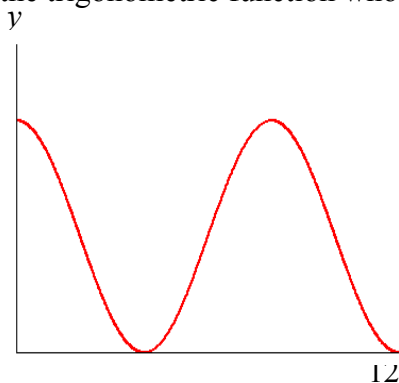
- the total number of cars that were sold in 10 years
  - the number of cars sold during the year 2000.
  - the number of years that have past since 1990 if a total of 10 million cars have been sold.
  - in 10 years,  $t$  cars were sold.
  - none of these
23. Write a polynomial equation, of *smallest degree*, for the function whose graph is shown below.



24. What happens to  $f(x) = \frac{4000 - x^2 - 4x^3}{2x^3 + x - 8000}$  as  $x \rightarrow +\infty$ ?

- $f(x) \rightarrow -\frac{1}{2}$
- $f(x) \rightarrow \frac{1}{2}$
- $f(x) \rightarrow -2$
- $f(x) \rightarrow 2$
- none of these

25. Find a formula for the trigonometric function whose graph is shown below.



**Part II:** Before you begin, spend a few minutes reading each question. Answer *only 4 questions* on this part. Each question is worth 15 points. Be sure you clearly indicate the questions you *do not* wish to be graded. Show all work.

1. (Divided as 3 points per row)

Copy the table below in your blue book. Fill in the blanks of the table for which you have sufficient information. Put a question mark (?) in any box for which there is not enough information to obtain an answer.

$x$	-4	-2	0	2	4
$f(x)$	2	-2	7	1	-5
$-f(-x) - 6$					
$f(2 - \frac{1}{2}x)$					
$2 - \frac{1}{2}f(x)$					

Copy the table below in your blue book. If  $g(x)$  is an odd function and  $h(x)$  is an even function, fill in the blanks of the table for which you have sufficient information. Put a question mark (?) in any box for which there is not enough information to obtain an answer.

$x$	-3	-2	-1	0	1	2	3
$g(x)$	-6	4	3	0			
$h(x)$	-6	-1	2	1			

2. (Divided as 4, 5, 2, 4)

- a. Given  $f(x) = \cos x$ , write an equation for a new cosine function,  $g(x)$ , that is a transformation of  $f(x)$  with the following properties, performed in the following order:
- the amplitude of the new function is 3,
  - the equation of the midline of the new function is  $y = 7$ ,
  - the graph of the new function makes one complete cycle in  $\frac{\pi}{3}$ ,
  - the graph of the new function has been shifted 3 units to the left.
- b. A racecar is traveling around a circular track. At any time  $t$ , the distance between the car and a line of bleachers is given by  $L(t) = 700 + 200\sin(\frac{2t}{3})$ , where  $L$  is in feet and  $t$  is in seconds.
1. Find the distance around the track.
  2. How long does it take the car to travel once around the track?
  3. Find the average speed of the car, in feet per second, for one lap around the track.
- c. Suppose there is a piece of gum stuck to an automobile tire. A function that represents the height of the gum from the ground is given by  $h(x) = 13\sin(4\pi t - \frac{\pi}{2}) + 13$ , where  $t$  is measured in seconds and  $h$  is measured in inches. Estimate when, during the first half second, the piece of gum is more than 9 inches from the ground. Round the time to the nearest tenth of a second.

- d. Find the *exact* time(s) during the first half-second that the piece of gum referred to in part (c) is more than 9 inches from the ground. Only an *algebraic* solution will be accepted.

3. (Divided as 1 point each)

Answer TRUE or FALSE

- If a population is decreasing by 20% per year, then in five years the population will be zero.
- Logarithmic functions grow at a constant rate.
- It is possible for the graph of a function to be increasing and concave down.
- If  $x$  and  $y$  are positive numbers, then  $3 \log x - \log y = \log \frac{x^3}{y}$ .
- The graphs of  $y = A \sin(Bx) + k$  and  $y = A \cos[B(x + \frac{\pi}{2})] + k$  are identical.
- For any value of  $\theta$ ,  $\sin^2(5\theta + 2) + \cos^2(5\theta + 2) = 1$ .
- A population that has 1000 members and decreases at a rate of 10% per year can be modeled as  $P = 100(0.10)^t$ .
- In general,  $f(g(x)) = g(f(x))$ .
- A function of the form  $y = mx + b$ ,  $m \neq 0$  has an inverse that is also a function.
- In general,  $f(a + b) \neq f(a) + f(b)$ .
- Every even function is invertible.
- On the interval  $[-2\pi, 2\pi]$ , the sine function is invertible.
- The  $y$ -intercept of the graph of a polynomial  $p(x)$  can be found by evaluating  $p(0)$ .
- The graph of a rational function may cross an asymptote.
- The function  $y = \sin(bx)$  has a period  $b$  times as large as the function  $g(x) = \sin x$ .

4. (Divided as 2, 2, 2, 3, 3, 3)

The population of a country has an annual continuous growth rate of 2.38%. There were 1,200,000 people in the country in 1996. In that same year, the country produces enough food for exactly 2,000,000 people. The food supply grows at a constant rate. Specifically, the number of people that can be fed each year increases by 15,000.

- Express the population  $P$ , as a function of  $t$ , the number of years since 1996.
- Express the number of people that can be fed,  $N$ , as a function of  $t$ , the number of years since 1996.
- Evaluate  $P(8)$  and interpret its meaning. Round your answer to the nearest integer.
- Find the average rate of change in the population from 1996 to 2004. Be sure to indicate units. Round your answer to the nearest integer.
- In what year* will the country's food supply first be unable to feed its population? Round your answer to the nearest year.
- Suppose the rate at which the food supply grows were doubled, to 30,000 people per year. Will this double the number of years until the country runs out of the food necessary to feed its population? Justify your answer.

5. (Divided as 3, 3, 6, 3)

In economics, the *demand* for a product is the amount of that product that consumers are willing to buy at a given price. The *supply* of a product is the quantity of that product that suppliers are willing to provide at a given price. Suppose both supply and demand can be modeled by linear functions.

A supplier of widgets will supply 490 widgets if the price is \$3.00, and will supply 850 widgets if the price is \$5.00. Consumers are willing to buy 180 widgets if the price per widget is \$1.00, and they will buy 170 widgets if the price is \$1.50 per widget.

- Find an equation for the quantity supplied,  $q_s$ , as a function of price,  $p$ .
- Find an equation for the quantity demanded,  $q_d$ , as a function of price,  $p$ .
- Equilibrium* is defined to be the price at which the quantity supplied equals the quantity demanded. Find the equilibrium price and the corresponding equilibrium quantity.
- Suppose a value added tax of 25 cents per widget is imposed on the *consumer*. This means that the consumer now must pay  $\$(p+0.25)$  per widget. Find the new equilibrium price (rounded to the nearest cent) and the corresponding equilibrium quantity (rounded to the nearest whole unit).

6. (Divided as 2, 3, 5, 5)

A cup of coffee contains 80 mg of caffeine, which leaves the body at a continuous rate of 17% per hour. I drank my first cup of coffee at 8:00 AM.

- Find a formula for the amount of caffeine in the body,  $Q$ , as a function of time,  $t$ , in hours.
- What is the half-life of caffeine?
- Assuming no other caffeine is ingested, what *percent* of the caffeine is in the body at 4:00 PM?
- If a second cup of coffee were ingested at 9:00 AM, what would be the total *amount* of caffeine in the body at 10:00 AM?

7. (Divided as 2, 3, 3, 4, 3)

It costs a company \$30,000 to begin the production of skateboards, plus \$3.00 for every skateboard produced. Let  $x$  be the number of skateboards produced by the company.

- Find a formula for  $C(x)$ , the total cost for the production of  $x$  skateboards.
- Find a formula for the company's average cost per unit,  $A(x)$ .
- Using a sentence or two, explain in *economic terms*, why the graph of  $A$  has the long term behavior it does.
- Find a formula for  $A^{-1}(x)$  and explain its *economic* significance.
- The company will make a profit if  $A(x) < \$5.00$ . Find the minimum number of units the company must produce in order to make a profit.