**NCC Precalculus Partnership Program**
Final Examination, 2005

Part I: Answer all 25 questions on this part. Each question is worth 2 points. Leave all answers in EXACT form, i.e., in terms of e, π, $\ln$, $\sqrt{}$, etc., unless otherwise instructed. No partial credit will be given.

1. Which of the following *does not* represent a function?

    a. The inverse of $g(\theta) = \sin\theta$, $-\infty < \theta < \infty$

    b. $\{(\pi,\pi), (6,9), (9,9)\}$

    c. $\{(x,y)\}$ where $y = \dfrac{1}{x}$

    d. $\{(x, f(x))\}$ where x is a cell phone number and $f(x)$ is the last digit of that number

    e. None of these

2. A function in which y is proportional to x is represented graphically by which of the line(s) below:
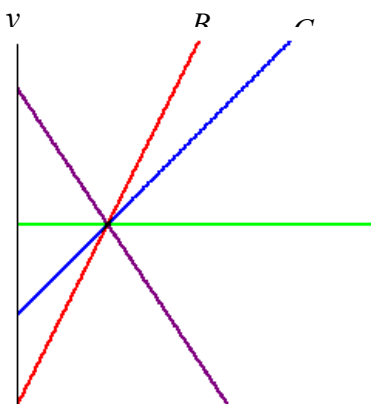

    a. A, B, C, and D b. B, C and D c. B and C d. only B e. none of these

3. Suppose $U = f(a)$ represents the number of air conditioners sold, in 10,000s, as a function of the amount spent on advertising, in $1000s. Interpret $f^{-1}(20) = 14$.

    a. When 20,000 air conditioners were sold, $14,000 was spent on advertising.

    b. When 140,000 air conditioners were sold, $20,000 was spent on advertising.

    c. When 20 air conditioners were sold, $14 was spent on advertising.

    d. When $200,000 was spent on advertising, 14,000 air conditioners were sold.

    e. None of these

4. Suppose the average rate of change of $y = f(x)$ between $x = -2$ and $x = 4$ is $-\dfrac{1}{3}$. If $f(4) = 12$, what is $f(-2)$?

5. What is the domain of the function $f(x) = \dfrac{x-9}{\sqrt{9 - x^4}}$?

6. If $g(x) = 3^x$, write an expression for $\dfrac{g(x+h) - g(x)}{h}$.

7. The graph of the linear function below shows a relationship between cups of tea consumed and hours of sleep obtained. The function is defined for $0 \le q \le 18$, however the graph only depicts values for $4 \le q \le 17$. Which of the following statements is (are) correct?

    I.   For every cup of tea consumed, two less hours of sleep are obtained.
    II. If you don't drink any tea, nine hours of sleep will be obtained.
    III. For every one less cup of tea consumed, one half-hour more sleep was obtained.


s, hours of sleep obtained

q, cups of tea consumed
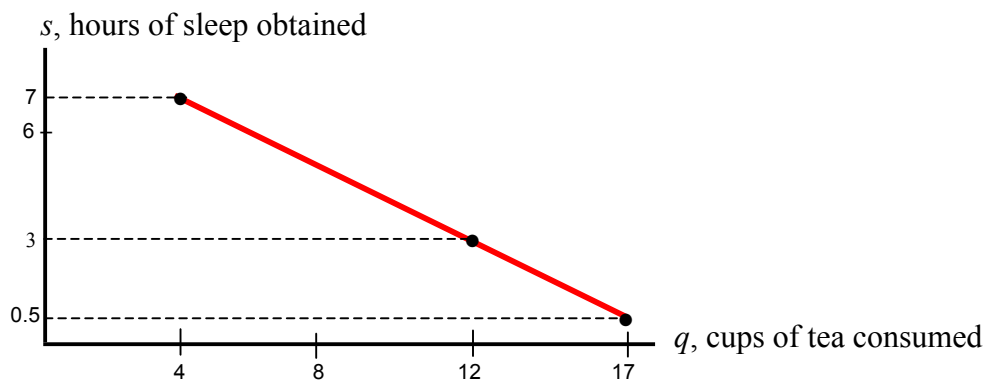
    a. only I   b. only II   c. only III   d. I and II   e. II and III

8. Write an equation for line l.

$y = \sin x + 1$
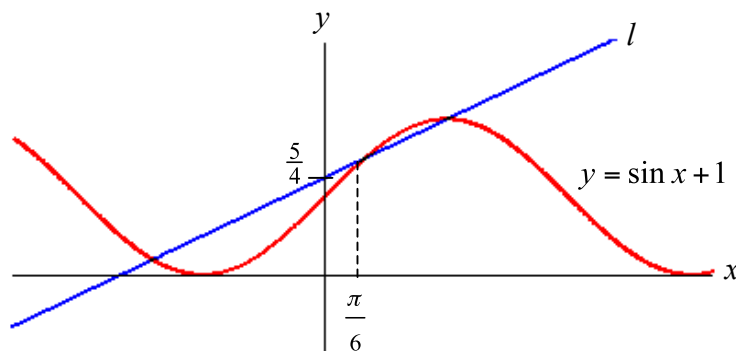
9. If $A = \ln x$ and $B = \ln y$, which is an expression for xy^2?

    a. $e^A + e^{2B}$ b. $\ln A + 2\ln B$ c. e^{A+2B}

    d. $\ln AB^2$ e. none of these

10. Cashews cost \$5.75 per pound and peanuts cost \$2.50 per pound. If you spend a total of \$32 on cashews and peanuts, express the number of pounds of cashews you can purchase, C, as a linear function of the number of pounds of peanuts you purchase, p.

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11. Find a formula for the trigonometric function whose graph is shown below.
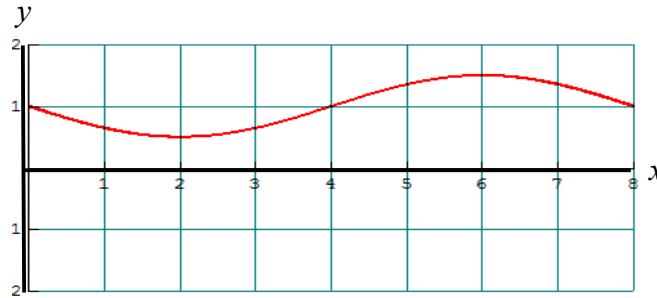
12. Consider the function defined by the table below.

x	-13	-12	-11	0	11	12	13
$h(x)$	a	b	c	0	$-c$	$-b$	$-a$

The function $h(x)$

    a.   is an decreasing function   d. is an even function

    b.   has a graph that is symmetric to the x-axis   e. an increasing function

    c.   has a graph that is symmetric to the origin

13. If $f(x) = 3e^{2x} - 1$, express f^{-1} as a function of x.

14. The functions $y = f(x)$ and $y = g(x)$ are defined by the table below. If $g(f(x)) = 0$, find x.

x	-2	2	5	6	11	12
$f(x)$	3	6	0	-2	5	11
$g(x)$	1	-2	6	0	3	5

15. During a hurricane, a brick breaks loose from the top of a chimney, 42 feet above the ground. As the brick falls, its distance from the ground after t seconds is given by $d(t) = -16t^2 + 42$. What is the domain of $d(t)$?

16. Suppose $C(x)$ represents the price, in dollars, of x pounds of coffee. Due to a drought in Brazil, the cost of coffee has since increased by 50%. How much will I pay for x pounds of coffee if I use a \$3.00 coupon to defray the cost?

    a. $C(1.5x) - 3$ b. $1.50C(x) - 3$ c. $0.5C(x) - 3$ d. $C(1.5x - 3)$ e. $C(0.5x) - 3$

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17. The equation of the graph shown below is $y = 3\cos B\theta$. Find B.
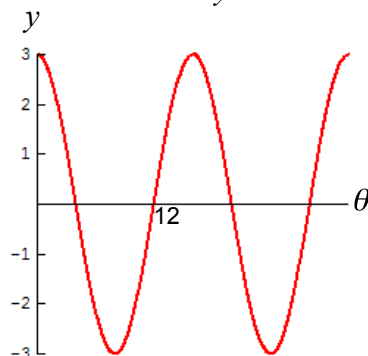
18. As $x \to \infty$ which of the following functions has the largest y value? Assume $a, b, c, d,$ and k are positive constants.

    a. $y = ax^3 + 1000x + \pi$ b. $y = b\sqrt{x} + 10^{203}$ c. $y = cx^{\pi} - 10^{203}$

    d. $y = dx^e + 1000$ e. $y = k\left(\frac{1}{2}\right)^x + \pi^{93}$

19. The amount of water in a barrel is always decreasing by a constant percent of its current value, i.e., the water leaks out at a continuous percent rate. Initially, there were 250 gallons of water in the barrel. After 4 hours, there were 213 gallons in the barrel. Express the number of gallons of water in the barrel, A, as a function of the elapsed time, t, in hours. *Round any constants to the nearest hundredth.*

20. Let $h(x) = \begin{cases} 3x + 4 & \text{for } x < -2 \\ -x^2 + 1 & \text{for } x \geq -2 \end{cases}$. What is the value of $h(-2)$?

21. Consider the graph of $f(t) = \cos t$. A new cosine curve, $g(t)$ is to be formed by transforming the graph of $f(t)$. The new graph is to have amplitude equal to 2, its midline is to have the equation $y = 3$, and it is to complete one cycle in $\dfrac{\pi}{2}$. Furthermore, it is to be shifted to the left by 5 units. Write an equation for the new function, $g(t)$.

22. The fourth degree polynomial $y = f(x)$ has a double zero at $x = -2$ and two other zeros, one at $x = 2$ and the other one at $x = 1$. If $f(0) = 24$, find an equation for the polynomial.

23. What is the limiting value of $f(x) = \dfrac{\sin x + 10^9}{-x + 1}$ as $x \to \infty$?

    a. $f(x) \to -1$ b. $f(x) \to 1$ c. $f(x) \to 10^9$

    d. $f(x) \to 0$ e. $f(x) \to \infty$

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24. The decay of the radioactive isotope carbon-14 can be modeled by $Q = Q_o e^{-0.00012t}$. How long will it take for any sample of carbon-14 to be reduced by 90% of its initial amount? *Round your answer to the nearest year.*

25. The graph of $y = f(x)$ is shown below. Sketch a graph of $g(x) = -f(-x) + 1$.
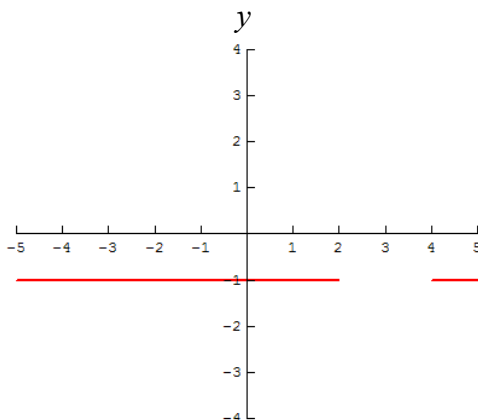
Part II: Before you begin, spend a few minutes reading each question. *Answer only 5 questions on this part.* Each question is worth 10 points. Be sure you clearly indicate the question you *do not* wish to be graded. Show all work.

1. (Divided as 2, 3, 5)

   A bank account earns a nominal interest of 4.5% per year, compounded continuously.

   a. By what percentage does the balance in the account increase over one year? That is, what is the effective annual yield (EAY)? *Round your answer to the nearest thousandth.*

   b. How long will it take for the balance in part (a) to triple? *Round your answer to the nearest tenth.*

   c. In another bank, the compounding period was daily. An initial balance of $4000 grew to $5049.29 in five years. What was the EAY? *Round your answer to the nearest thousandth.*

2. (Divided as 2, 2, 2, 4)

   A colony of mice has annual continuous growth rate of 1.56%. There were 700,000 mice in the colony in 1995, when $t = 0$. In that same year, a hawk breeding program was begun. When the hawk population was large enough, it would be introduced into the region that the mice inhabited in order to eradicate the entire mouse population in that one year. The population of hawks grew over time as the table below shows. It is known that, on average, each hawk consumes 1200 mice per year.

Year	0	1	2	3	4	5	6	7	8	9
Hawks	100	160	190	255	280	370	380	450	512	547

   a. Express the population of hawks H, as a best fitting linear function of t, the number of years since 1995. *Round all constants to the nearest whole number.*

   b. Find the value of $H(10)$ and interpret its meaning.

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c. Write an equation, $M(t)$, that models the growth of the mouse colony as a function of t, the number of years since 1995.

d. Will the hawk population ever grow large enough to eradicate the entire mouse population in one year? If so, when? If not, why not? *Carefully support your answer.*

3. (Divided as 2, 3, 2, 3)

   The volume of pollutants (in millions of cubic feet) in a particular reservoir is given by $P(t) = 65 + 350t^2 + 6t^3$, where t is in years. The volume of fluid (in millions of cubic feet) in the reservoir (water and pollutants) is given by $V(t) = 120t^3 + 3t^2 + 275t + 650$. Let $C(t)$ be the fraction of the total volume that is exclusively pollutants.

   a. Write an expression for $C(t)$ in terms of t.

   b. What is the average rate of change of pollutants between $t = 0$ and $t = 5$? *Be sure to include units in your answer.*

   c. In year $t = 0$, what percentage of the reservoir's total volume consists of pollutants?

   d. If these trends were to continue for many years, what percentage of the reservoir's total volume would eventually consist of pollutants?
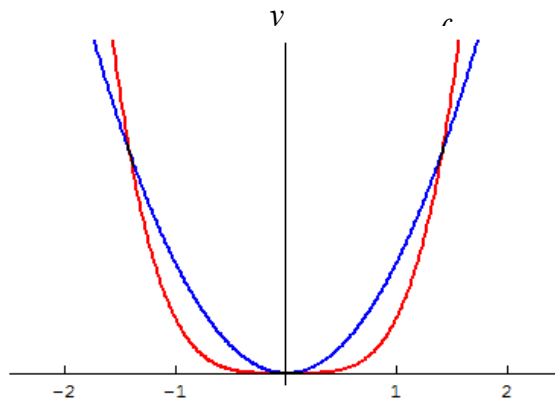
4. (Divided as 3, 3, 2, 2)
   A. A rational function has the following features:

   - vertical asymptotes at $x = 2$ and $x = -2$
   - a horizontal asymptote at $y = 4$
   - zeros at $x = -1$ and $x = 1$
   - a vertical intercept at $y = 1$
   - when $x > 2$ and when $x < -2$, $f(x) > 4$.

   1. Sketch the graph of the function.

   2. Find an equation for this function.

   B. The figure below shows the graphs of $f(x) = ax^p$ and $g(x) = bx^q$.

   1. Which is larger, p or q? Explain your answer in a short sentence.

   2. Which is larger, a or b? Explain your answer in a short sentence.

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5. (Divided as 1, 2, 2, 3, 2)

   A. Two weights, W_1 and W_2, are suspended from the ceiling by springs. At time $t = 0$ (t in seconds) the weights are set in motion and begin bobbing up and down. Eventually, the oscillation of both weights dies down and they cease bobbing. The following equations describe the distance of each weight from the ceiling as a function of time:
   $$d_1 = 6 + 4\cos(\pi t)e^{-0.2t} \quad \text{and} \quad d_2 = 5 + 3\cos(2\pi t)e^{-0.5t}.$$

      1. Initially, how far are W_1 and W_2 from the ceiling?

      2. When W_1 and W_2 stop bobbing, how far are W_1 and W_2 from the ceiling?

      3. When, *to the nearest tenth of a second*, are the two weights furthest apart?

   B. Starting one year before an election, a study was conducted to determine the popularity of the candidates. In the table below, $A(t)$ represents the percent of the electorate that favor candidate A, t months after the start of the study.

t	0	3	6	9	12
$A(t)$	22	42	62	42	22

      1. Express A as a trigonometric function of t. *Be sure to express your answer using exact values*.

      2. Support for candidate B is given by $B(t) = 31 + 15\sin\left(\dfrac{\pi}{6}t\right)$ and support for candidate C is given by $C(t) = 42 - 20\sin\left(\dfrac{\pi}{6}t + \dfrac{\pi}{2}\right)$. For what value(s) of t, *to the nearest integer*, are candidates B and C tied for support?

6. Divided as 1 point each

   Answer each sentence with the words "always true", "sometimes true", or "never true".

   a. If $y = f(w)$ then $f(w + z)$ is equal to $f(w) + f(z)$.

   b. If the graph of the function $f(x)$ is concave up and $k < 0,$ then the graph of $f(kx)$ is concave down.

   c. A quadratic equation can be expressed as the sum or difference of two squares.

   d. The equation $23e^{2x} = x + 1$ can be solved for x by taking the natural logarithm of both sides of the equation.

   e. The function $f(t)$ and $f(kt)$ have the same zeros.

   f. Two investments are compounded continuously. If the interest rates are the same, the time it takes for a \$10,000 investment to triple is the same time as it takes a \$75 investment to triple.

   g. A cubic equation crosses the x-axis three times.

   h. If $f(x) = \ln x$ and $g(x) = -e^x$, then $f(g(x))$ exists.

   i. The graph of an exponential function can intersect the graph of a quadratic function three times.

   j. The function $f(x)$ and $f(x) + c$ have the same domain.

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