2006 Final Exam
Answers, Part I

1. 13
2. C
3. 13.86%
4. B
5. $h(t) = -16(t-3)^2 + 148$
6. 5
7. $a - b$
8. B
9. $h(x) = f(x+4) + 2$
10. $x = e^n + a$
11. D
12. 5 people
13. $y = 2\sin\left(\frac{2}{3}\pi t\right) + 1$
14. C
15. D
16. 5000 miles
17. $y = -\frac{1}{8}(x+2)^2(x-4)$
18. $-\frac{1}{2}$
19. C
20. B
21. $\frac{\ln(e+2)}{e+1}$
22. B
23. E
24. B
25. C

2006 Final Exam
Solutions, Part II

1. a. $A(t) = (6+3t)(4+t) = 3t^2 + 18t + 24$

   b. $A(2) = 72\,\text{mi}^2$

   c. $3t^2 + 18t + 24 = 240 \rightarrow 3t^2 + 18t - 216 = 0 \rightarrow (t+12)(t-6) = 0 \rightarrow$ Domain: [0, 6] min.

2. a. $h(0) = f(0) - g(0) \rightarrow h(0) = -4 - 7 = -11.$

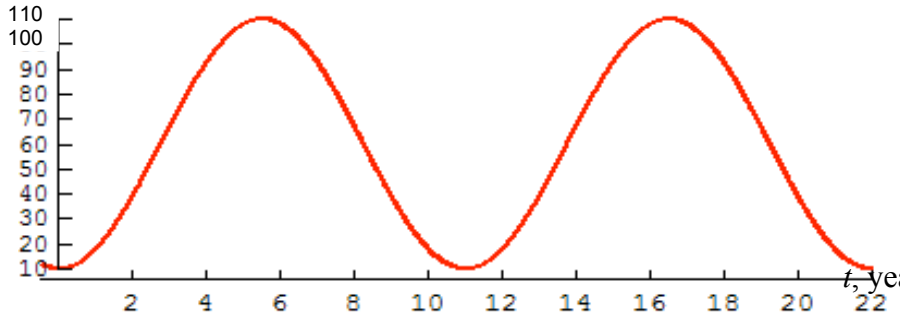
   b. $0 = f(x) - g(x) \rightarrow f(x) = g(x) \rightarrow x = -1$ and $x = 3.$

   c. $h(x) = \frac{11}{3}(x+1)(x-3).$

   d. $x = -1$ and $x = 3.$

3. a. Period = 11 years.

   b. N, number of sunspots



   c. $N(t) = 60 - 50\cos\left(\frac{2\pi t}{11}\right).$

   d. In 2006, one would expect ≈ 67 sunspots. (Exact value is 67.11 which is not in the domain).

4. a. $0.38 = b^{45} \rightarrow b = 0.9787$, so $N(t) = 5.64(0.9787)^t$.

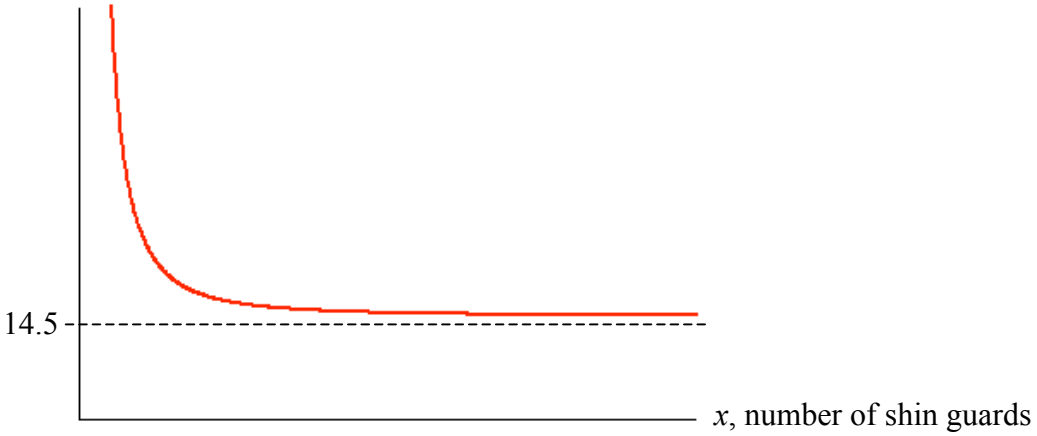
   b. $N(57) = 5.64(0.9787)^{57} \rightarrow$ In 2007, 1.653 million farms will exist.

   c. $F = 5.64(0.9787)^t \rightarrow \log\left(\frac{F}{5.64}\right) = t\log 0.9787 \rightarrow t = \frac{\log\left(\frac{F}{5.64}\right)}{\log 0.9787}.$

5. a. $C(x) = 10,000 + 14.5x.$

b. $\dfrac{C(x)}{x}$ is the average cost per shin guard per week.

c. $\dfrac{C(x)}{x}$, average cost per shin guard per week.



d. $\dfrac{C(x)}{x} < k \rightarrow \dfrac{10,000 + 14.5x}{x} < k \rightarrow x > \dfrac{10,000}{k - 14.5}.$

e. $P^{-1}(x)$ is the number of shin guards that has to be produced to be sold if a profit of $\$P$ was made.

6. a. $\log(x-15) + \log x = 2 \rightarrow \log[x(x-15)] = 2 \rightarrow x^2 - 15x = 10^2$

$x^2 - 15x - 100 = 0$

$(x-20)(x+5) = 0$

$x = 20.$

b. $\sin^2 x = \sin x \cos x \rightarrow \sin^2 x - \sin x \cos x = 0$

$\sin x(\sin x - \cos x) = 0$

$\sin x = 0$ and $\sin x = \cos x$

$x = 0, \pi$ and $x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}.$

7. a $S = f(w) = 0.57w + 23.96.$

b. $f(135) = 100.91\,\text{ft}.$

c. $0.57w + 23.96 = 200 - (50 + 6) \rightarrow 0.57w + 23.96 = 144$

$w = 210.6$ lbs.

d. No, since the weight of 210.6 lbs is outside the known values. It is possible that the stretch would be more variable at higher weights.