NCC Pre Calculus Partnership Program
Final Examination, 2006

Part I: Answer all 25 questions on this part. Each question is worth 2 points. Leave all answers in EXACT form, i.e., in terms of e, π, ln, $\sqrt{\ }$, etc., unless otherwise instructed. No partial credit will be given.

1. A sinusoidal curve with an amplitude of 4 has a minimum value of 5. What is its maximum value?

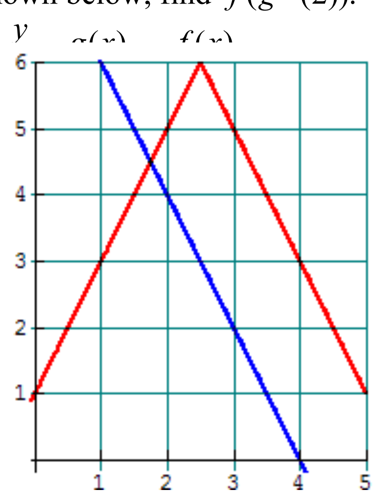
2. Solve for x, if $8y = 3e^x$.
   a. $x = \ln 8 + \ln 3 + \ln y$
   b. $x = \ln 3 - \ln 8 + \ln y$
   c. $x = \ln 8 + \ln y - \ln 3$
   d. $x = \ln 3 - \ln 8 - \ln y$

3. Suppose you invest p dollars in an account that is compounded continuously. What interest rate is needed for this investment to double in five years? *Round your answer to the nearest hundredth of a percent.*

4. Suppose you go to your favorite pizzeria and a cheese and pepperoni pizza is brought to your table right out of the oven. If the temperature of the pizza, H, is a function of time, t, that is $H = f(t)$, and the temperature of the pizza changes exponentially then
   a. f is increasing and its graph is concave up
   b. f is decreasing and its graph is concave up
   c. f is increasing and its graph is concave down
   d. f is decreasing and its graph is concave down
   e. f is decreasing linearly

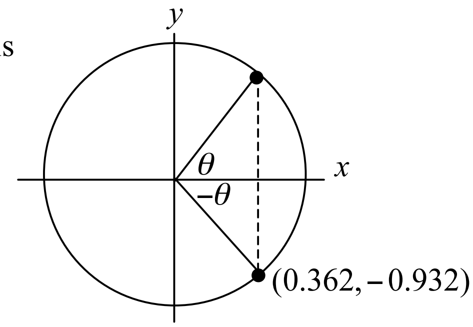
5. Ruth, the leadoff batter on the Nassau All Star baseball team hit a high fly to right field. The ball was 4 feet above the ground when she hit it. Three seconds later, it reached its maximum height, 148 feet. If the path of the ball is parabolic, write a quadratic function in vertex form, that expresses the height of the ball h, as a function of time, t.

6. For the functions f and g shown below, find $f(g^{-1}(2))$.



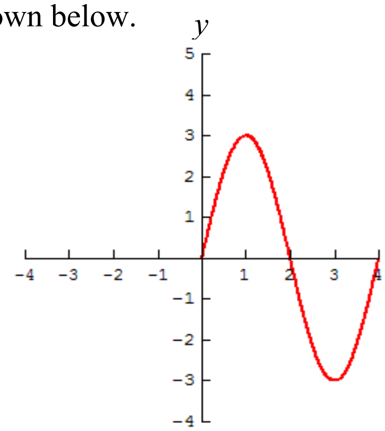
7. Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $f(-2) = a$ and $g(2) = b$, what is the value of $f(2) + g(-2)$?

8. Using the diagram to the right, angle θ measured in radians is

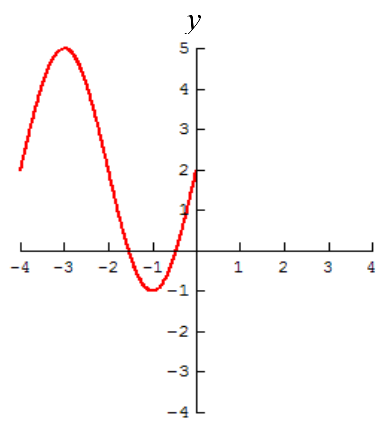


   a. 0.37
   b. 1.20
   c. -1.20
   d. 68.78
   e. 0.88

9. The graph of $y = f(x)$ is shown below.



The graph of $h(x)$, shown below, is a transformation of the graph of $f(x)$. Write a formula for $h(x)$ in terms of $f(x)$.

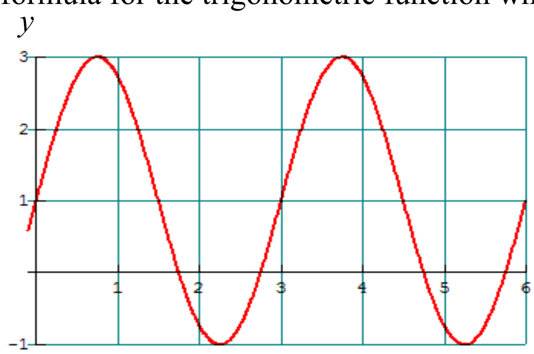


10. Solve the following equation for x: $\ln(x - a) = n$

11. Which function below is increasing on the domain of all real numbers?
   a. $f(x) = -2e^x$
   b. $f(x) = 2x^2$
   c. $f(x) = 2^{5e}$
   d. $f(x) = \dfrac{3}{1+e^{-x}}$
   e. All of the above

12. An organization determines that the cost per person for chartering a bus is given by the formula $C(p) = \dfrac{100+5p}{p}$, where p is the number of people chartering the bus, such that $0 \le p \le 40$ and $C(p)$ is in dollars. Determine $C^{-1}(25)$. *Be sure to include units*

13. Find a possible formula for the trigonometric function whose graph is shown below.



14. Given the information in the table shown below, which function is changing exponentially? Assume all constants are greater than one.

a. $f(x)$ b. $g(x)$ c. $h(x)$ d. $k(x)$

x	$f(x)$	$g(x)$	$h(x)$	$k(x)$
1	a	$b+4$	c	$\frac{d}{2}$
2	$3a$	$b+8$	c^2	$\frac{d}{4}$
3	$6a$	$b+12$	c^3	$\frac{d}{6}$
4	$9a$	$b+16$	c^4	$\frac{d}{8}$

15. The temperature, H, of the surface of the water in a pond is a function of time, t, the number of hours since 6am such that $H = 20 - 3\cos\left(\frac{\pi}{12}t\right)$, where $0 \le t \le 12$. A formula for the time t, as a function of the temperature can be expressed as

a. $t = \dfrac{20-H}{3\cos\left(\frac{\pi}{12}\right)}$

b. $t = \frac{\pi}{12}\left[\arccos\left(\frac{20-H}{3}\right)\right]$

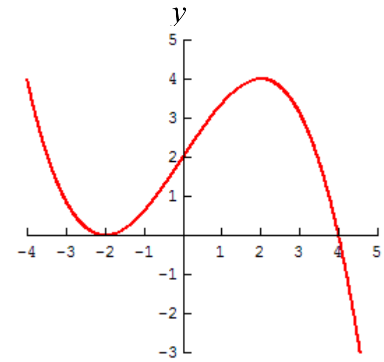
c. $t = \frac{\pi}{12}\left[\cos\left(\frac{20-H}{3}\right)\right]$

d. $t = \frac{12}{\pi}\left[\arccos\left(\frac{20-H}{3}\right)\right]$

e. $t = \frac{12}{\pi}\left[\cos\left(\frac{20-H}{3}\right)\right]$

16. The weight, W, in pounds, of an astronaut varies inversely as the square of the distance from the center of the Earth, d, in miles. If the constant of proportionality is 2.88×10^{9} and the astronaut weighs 115.2 pounds, d miles from the center of the Earth, find d.

17. Write an equation of the polynomial of the smallest degree whose graph is shown below.



18. Let $f(x)=\dfrac{3x^3+8}{2-6x^3}$. As the value of x gets very large $(x\to\infty)$, what value does $f(x)$ approach?

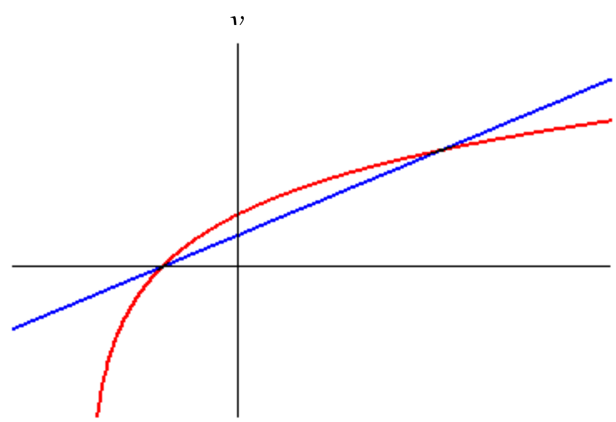
19. Let $f(x)=ab^x$, $b>0$. Then $\dfrac{f(x+h)}{f(x)}$ is equal to

   a. h
   b. $b^{x+h}-b^x$
   c. b^h
   d. a

20. Which of the following statements is *false*?

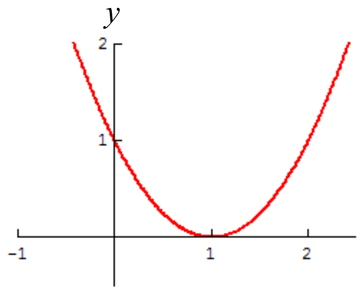
   a. A linear function has a constant slope.
   b. Exponential functions grow at a constant rate.
   c. For all x, $\ln e^x = x$.
   d. A linear function can intersect an exponential function two times.
   e. Logarithms are exponents.

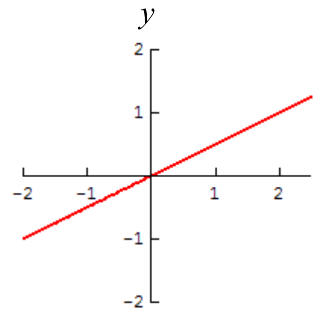
21. Find the *exact* value for the slope of line L.



$(x+2)$

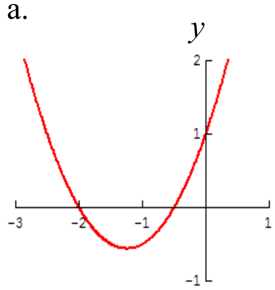
22. Consider the graphs of $f(x)$ and $g(x)$ shown below.



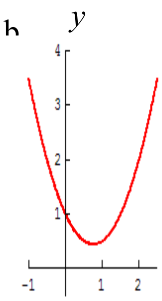


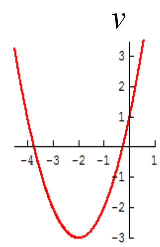
Which of the following graphs represents $f(x) + g(x)$?

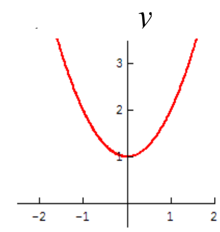
a.



b



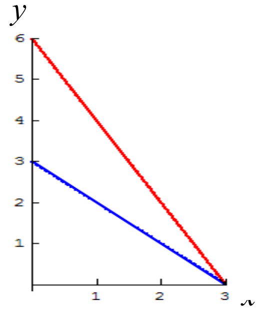




23. Which of the following functions has its domain identical to its range?
   a. $g(x) = \sqrt{x}$
   b. $h(x) = \begin{cases} |x| & -4 \le x < 0 \\ -x & 0 \le x \le 4 \end{cases}$
   c. $f(x) = x^3$
   d. $j(x) = \dfrac{1}{x}$
   e. all the above

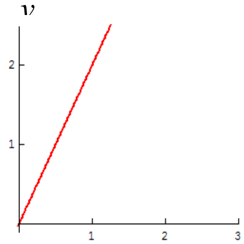
24. If $f(x) = e^x$, which of the following statements is *false* about $f^{-1}(x)$?
   a. $f^{-1}(x)$ is an increasing function.
   b. $f^{-1}(x)$ is symmetric about the origin.
   c. $f^{-1}(x)$ is continuous on its domain
   d. $f^{-1}(x)$ is has a vertical asymptote.

25. Consider the two linear functions whose graphs are shown below.

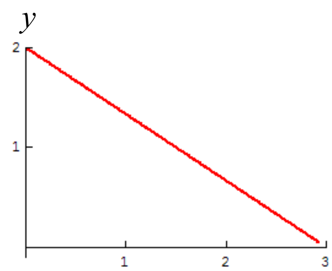


On the interval $0 \le x \le 3$, which of the following graphs represents the larger function divided by the smaller function?

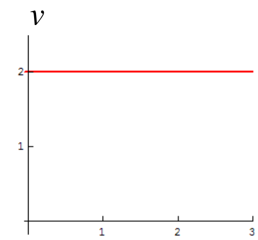
(a)



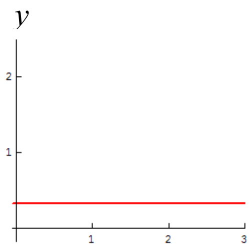
(b)



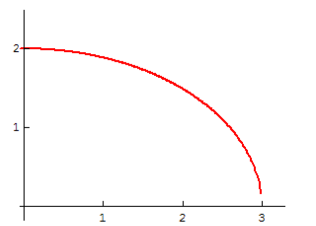
(c)



(d)



(e)

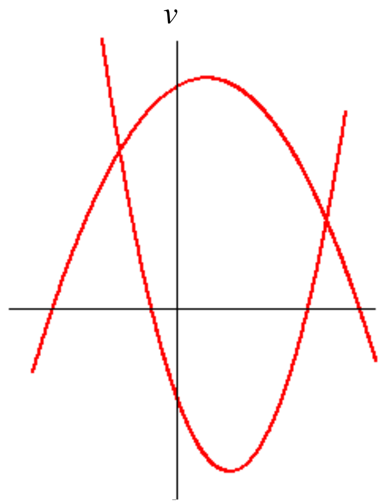


Part II: Before you begin, spend a few minutes reading each question. Answer *only 5 questions* on this part. Each question is worth 10 points. Be sure you clearly indicate the questions you *do not* wish to be graded. Show all work.

1. (2, 2, 6)
   a. A satellite camera aboard a rocket takes rectangular shaped picture of the Earth. The smallest region that can be photographed is a rectangle whose length is 6 miles and whose width is 4 miles. As the rocket rises, the length of the picture increases at a rate of 3 mi/min and the width of the picture increases at the rate of 1 mi/min. Write a function $A(t)$, that represents the area photographed as a function of time, t, the number of minutes after the camera begins taking pictures.
   b. How large is the photographed rectangular region 2 minutes after the camera begins taking pictures? Be sure to include units in your answer. *Only an algebraic solution will be accepted.*
   c. As the rocket rises, the area of the rectangle being photographed increases until the picture becomes too distorted to be useful. If the maximum area for clarity is $240\,\text{mi}^2$,

what is the domain of $A(t)$? Be sure to include units in your answer. *Only an algebraic solution will be accepted.*

2. (2, 2, 4, 2)
Consider the graphs of functions $f(x)$ and $g(x)$ shown below.



a. If $h(x) = f(x) - g(x)$, what is the value of $h(0)$?
b. What value(s) of x satisfy the equation $h(x) = 0$?
c. If $h(x)$ is quadratic, write an equation for $h(x)$.
d. If $k(x) = g(x) - f(x)$, for what value(s) of x is $h(x) = k(x)$?

3. (2, 2, 4, 2)

For several hundred years, astronomers have kept track of the number of sunspots that occur on the surface of the sun. Assume that the number of sunspots varies sinusoidally as a function of the year. There were 18 completed cycles between 1750 and 1948. In one cycle, the minimum number of sunspots was 10 per year and the maximum number of sunspots was 110 per year. Assume the first minimum occurred in 1750.

a. What is the period of the sunspot cycle?
b. Sketch a graph of two sunspot cycles, starting in 1948. Be sure to label the graph.
c. Write an equation expressing the number of sunspots per year, N, as a function of the year, t.
d. If this pattern continues, how many sunspots would you expect in 2006? *Round your answer to the nearest whole number.*

4. (5, 2, 3)

The number of farms in the United States has declined since 1950. In 1950, when $t = 0$, there were 5.64 million farms. In 1995, the number of farms had decreased by 62%. Assume the number of farms decreased by a *constant percent* per year.

a. Let $t = 0$ be the year 1950. Find a function $N(t)$, that describes the number of farms remaining after t years.
b. How many farms can expect to be in existence in 2007?

c. At this rate, in what year will F farms remain? *Your answer will contain the constant, F.*

5. (1, 2, 2, 3, 2)

The All Star Sporting Goods Company sells soccer shin guards. It costs \$14.50 to produce each pair of shin guards, and the company's weekly overhead cost is \$10,000.

a. Express the weekly cost equation $C(x)$, as a function of the number of shin guards produced, x.

b. Carefully interpret the expression $\frac{C(x)}{x}$, *in terms of this problem.*.

c. Sketch $y = \frac{C(x)}{x}$. Be sure to label the axes and note any interesting features of the graph.

d. The company will make a profit, $P(x)$, if the average cost of its shin guards is less than \$$k$ each. How many shin guards must be sold to insure that the company will make a profit? *Your answer will be in terms of x and k.*

e. *In terms of this problem*, what is the practical significance of $P^{-1}(x)$?

6. (5, 5)

Solve each equation for x. *Only algebraic solutions with exact values* will be given full credit. All work must be shown to receive partial credit.

a. $\log(x-15) + \log x = 2$

b. $\sin^2 x = \sin x \cos x \quad 0 \le x < 2\pi$

7. (2, 2, 3, 3)

A 50 ft long bungee will stretch a certain amount, which depends on how much the person jumping weighs. The following table tells how much a bungee cord will stretch for certain weights. The stretch is in addition to the original 50 ft length of the bungee cord. Assume that the stretch distance is a linear function of the weight of the person.

Weight (lbs)	Stretch (ft)
100	80.9
110	86.7
120	92.4
130	98.1
140	103.7

a. Write an equation that will predict the stretch distance for a given weight.

b. Use your equation to find the stretch distance for a person weighting 135 lbs.

c. If the concrete is 200 ft below the point where the bungee cord is attached, what is the heaviest "safe" weight for a 6 ft tall jumper? Assume the bungee cord is attached to the jumper's ankles.

d. If you were the person making the jump in part (c), would you feel confident completing the jump safely? Explain your answer using mathematical concepts.