2007 Final Examination
Answers, Part I

1. $C = f(d) = 540 + 0.35d$

2. -15

3. A

4. $[12, \infty)$ or $x \geq 12$

5. $h^{-1}(6) = -\frac{1}{3}$

6. B

7. B

8. C

9. D

10. C

11. $4ah + 2h^2 - 3h$

12. $100

13. $x = \frac{\log 9}{\log 18}$ or $x = \frac{\ln 9}{\ln 18}$

14 20

15. B

16. C

17. B

18. $a = -2$

19. A

20. $x = \frac{101}{3}$

21. D

22. $y = (4/3)x$ and $y = 2x - 6 \rightarrow (4/3)x = 2x - 6$. Solving gives $x = 9$ *and* $y = 12$, so the third line has the equation $(y - 12) = -2(x - 9)$ or $y = -2x + 30$. This line intersects the y-axis when $x = 0 \rightarrow y = 30$.

23. $f(x) = 400(1 + r)^t$

$f(3) = 400(1 - 0.5)^t = 50$, so

$g(t) = 50 = 4(1 + r)^3$

$12.5 = (1 + r)^3 \rightarrow 2.32 = 1 + r$.

Thus, $g(t) = 4(2.32)^t = 4(12.5)^{\frac{t}{3}}$.

24. $\ln y = x + \ln(x + 1) + \ln c$, so

$y = e^{x + \ln(x+1) + \ln c} = e^x e^{\ln(x+1)} e^{\ln c}$, so

$y = e^x (x + 1)(c)$.

25. $y = k(x + 11)(x + 6)(x + 1)$.

If $x = -12$ and $y = -33$,

$-33 = k(-1)(-6)(-11) = -66k$

$k = 1/2$.

So, $(1/2)(x + 11)(x + 6)(x + 1)$

26. $2 = e^{5.78k} \rightarrow k = (\ln 2 / 5.78) \approx 0.1199$

$3 = e^{\frac{\ln 2}{5.78}t} \rightarrow \ln 3 = 0.1199t$

$t = (\ln 3 / 0.1199)$. So, $t \approx 9.16$ months.

27. $H(t) = 4\cos Bt + 50$

and also, $60 = \frac{2\pi}{B} \rightarrow B = \frac{\pi}{30}$

Therefore, $H(t) = 4\cos\left(\frac{\pi}{30}t\right) + 50$.

2007 Final Exam
Solutions, Part II

1. a. Using a regression utility, $N(t) = -6.3t + 249.3$.
   b. $N(30) = 60$.
   c. $200 = -6.3t + 249.3 \rightarrow t \approx 8,$ so about 8:08.
   d. $0 = -6.3t + 249.3 \rightarrow t \approx 40,$ so 8:40.
   e. The answers do not agree. It is not correct to use the model since the time is too far removed from the data.

2. a. $P(t) = 4(2)^t$

   b. $P(14) = 4(2)^{14} \approx 65{,}536$ tribbles

   c. $\dfrac{7.8\times 10^6}{0.25} = 31{,}200{,}000$ tribbles can fit on the ship, so,

$31{,}200{,}000 = 4(2)^t$

$7{,}800{,}000 = 2^t$

$\ln(7{,}800{,}000) = t\ln 2 \rightarrow t = \dfrac{\ln(7{,}800{,}000)}{\ln 2} \approx 23$ days

   d.
$$E(t) = E_0 b^t$$
$$4 = E_0 b^{10} \text{ and } 972 = E_0 b^{15} \rightarrow b = 3$$
$$4 = E_0 3^{10} \rightarrow E_0 = 0.00006774$$
Therefore $E(t) = 0.00006774(3)^t$.

   e. We want $\dfrac{P(t)}{(100)} = E(t)$

$$\frac{4(2)^t}{100} = 0.00006774(3)^t$$
$$590.4930617(2)^t = 3^t$$
$$\ln 590.4930617 + t\ln 2 = t\ln 3$$
$$\ln 590.4930617 = t(\ln 3 - \ln 2)$$
$$t = \frac{\ln 590.4930617}{(\ln 3 - \ln 2)} \approx 16 \text{ days.}$$

3. a.
   i. $d + a$
   ii. $d - a$
   iii. $P = \dfrac{2\pi}{\frac{n}{b}} = \dfrac{2\pi b}{n}$.
   iv. $\dfrac{60}{\frac{2\pi b}{n}} = \dfrac{30n}{\pi b}$.

   b.
   i. $T = -15\cos\left(\dfrac{\pi}{12}t\right) + 45$
   ii. $T = -15\cos\left[\dfrac{\pi}{12}(t-4)\right] + 45$
   iii. $T = -15\cos\left(\dfrac{\pi}{12}t\right) + 35$

4. a. $f(x) = a(x-h)^2 + k$

$6 = a(0-1)^2 + 1 \rightarrow a = 5$

So, $f(x) = 5(x-1)^2 + 1 = 5x^2 - 10x + 6$.

b. 1. $(-2,10)$
2. $(9,-1)$
3. $(-3,-3)$
4. $(-2,1)$
5. $(3,2)$

5. a. $M = 60 - 3t$ and $P = 75 - 4t$

$60 - 3t = 75 - 4t \rightarrow t = 15$ minutes.

b. $0 = 75 - 4t \rightarrow t = 18.5$

$M = 60 - 3(18.75) = 3.75 \approx 4$ hot dogs.

c. $A = A_0 e^{-0.35t}$

$0.20 = e^{-0.35t}$

$t = \dfrac{\ln 0.20}{-0.35} \approx 4.6$ days.

6. a. $A = \dfrac{300 + 5d}{d} = \dfrac{300}{d} + 5$

b. As distances increases, the \$300 flat fee becomes less and less significant and the cost per mile fee is dominated by the per mile fee of \$5.00.

c. $A^{-1} = \dfrac{300}{A-5}$ represents the distance travelled for a given average cost per mile.

d. $d = A^{-1} > \dfrac{300}{8-5} = 100$ miles.

7. a.- *iii* b.- *v.* c.- *i.* d.- *iv.* e.- *vi.*