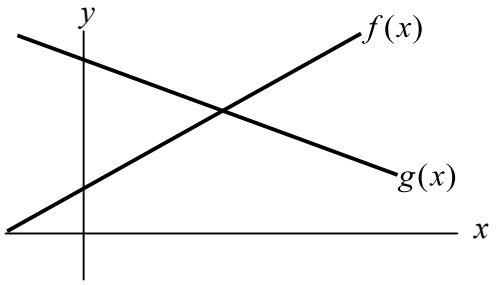
NCC Precalculus Partnership Program
Final Examination, 2007

Part I: Answer all 10 questions in this section of part I. Each question is worth 1 point. Leave all answers in EXACT form, i.e., in terms of e, π, ln, $\sqrt{\ }$, etc., unless otherwise instructed. No partial credit will be given.

1. A machine that manufactures compact discs costs \$540 to initialize plus \$0.35 to manufacture each disc. Write an equation that represents the total cost, *C*, of manufacturing *d* discs.

2. Find the average rate of change of $f(x) = x - 2x^2$ with respect to x between $x = 1$ and $x = 7$.

3. The graphs of $f(x)$ and $g(x)$ are shown below. If $h(x) = f(x) - g(x)$, then the slope of the graph of $h(x)$ is
   a. positive
   b. negative
   c. zero
   d. undefined



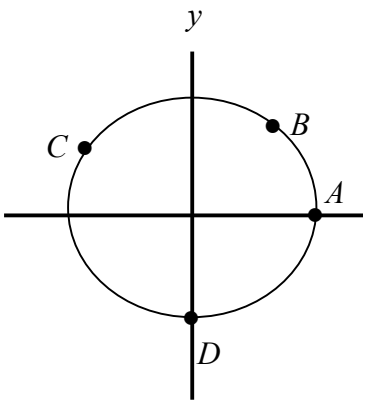
4. What is the domain of the function $f(x) = \dfrac{1}{\sqrt{x-12} + \sqrt{5}}$?

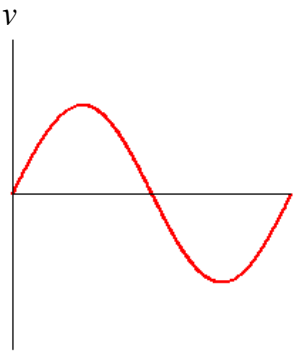
5. Given $h(x) = 3x + 7$, evaluate $h^{-1}(6)$.

6. In which of the following relations is *y not* a function of *x*?
   a. $\{(5, 4), (3, 1), (-5, 4), (-3, 1)\}$
   b. $y^2 = x + 1$
   c. $y = \dfrac{3}{x^2 - 5x}$
   d. The conversion of a student's numerical grade into a letter grade as shown in the table below.

90 –100	A
85 – 89	B+
80 – 84	B
75 – 79	C+
70 – 74	C
65 – 69	D+
60 – 64	D
Below 60	F

7. Consider the graphs shown below.





Point C on the unit circle might correspond to which point on the sine curve?

a. a b. b c. c d. d e. a value not shown

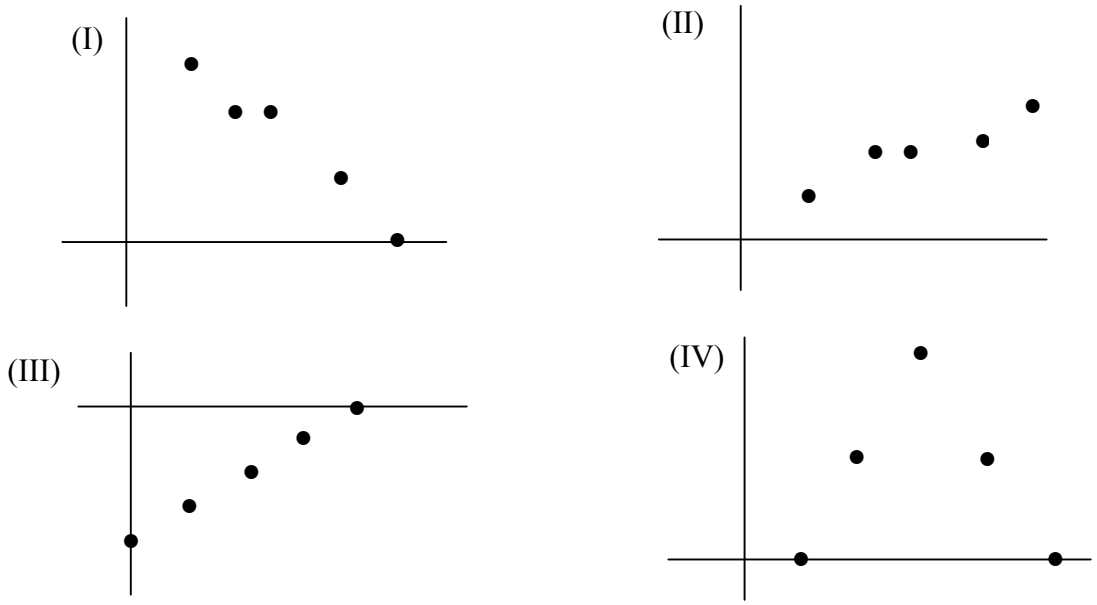
8. If $C(n)$ is the cost of producing n items, then the function $f(n) = \dfrac{C(n)}{n}$ represents

   a. The average rate of change of $C(n)$
   b. The average cost for producing n items
   c. The average cost for producing one item
   d. The long term cost for production
   e. None of these

9. Let $I(t)$ represent the number of iPods, in thousands, sold t years after 2004. Assuming that $I(t)$ is invertible, what is the meaning of $I^{-1}(24)$?

   a. The number of iPods that will be sold in 2028.
   b. The year 2028.
   c. There will be 24,000 iPods sold in 24 years.
   d. The number of years that have elapsed since 2004 if 24 thousand iPods have been sold.
   e. None of these

10. In the figures below, order the correlation coefficients *r*, from smallest to largest.
    a. (III), (II), (I), (IV)
    b. (IV), (II), (I), (III)
    c. (I), (IV), (II), (III)
    d. None of the above



Answer all 11 questions in this section of part I. Each question is worth 2 points. Leave all answers in EXACT form, i.e., in terms of $e, \pi, \ln, \sqrt{\ }$, etc., unless otherwise instructed. No partial credit will be given.

11. If $f(x) = 2x^2 - 3x$, find $f(a+h) - f(a)$ in simplest terms.

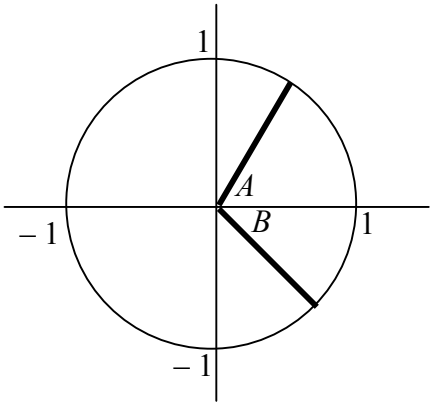
12. Elizabeth's great-grandmother invested money 100 years ago that grew at 5% annual interest compounded quarterly. The account balance is now \$14,388.41. *To the nearest dollar*, what was the great-grandmother's initial deposit?

13. Solve for the *exact* value of x: $\left(3^x\right)\left(6^x\right) = 9$.

14. Use the table below to find the value of $4f(2x+1)$ when $x = -2$.

x	-4	-3	-2	-1	0
$f(x)$	-6	5	2	3	-2

15. Which one of the following could be approximate values for the sine and cosine of angles A and B in the figure shown below?



a. $\sin A \approx 0.5,\ \cos A \approx 0.85,\ \sin B \approx -0.7,\ \cos B \approx 0.7$

b. $\sin A \approx 0.85,\ \cos A \approx 0.5,\ \sin B \approx -0.7,\ \cos B \approx 0.7$

c. $\sin A \approx 0.5,\ \cos A \approx 0.85,\ \sin B \approx 0.7,\ \cos B \approx 0.7$

d. $\sin A \approx 0.85,\ \cos A \approx 0.5,\ \sin B \approx 0.7,\ \cos B \approx 0.7$

16. Suppose $h(x) = f(g(x))$ and $f(x) = k$ and $g(x) = ax + b$, where k, a, and b are constants. Then $h(x)$ is

a. a linear function

b. a quadratic function

c. a constant function

d. a function whose classification cannot be determined from the given information.

17. Let $y = f(x)$ be a linear function containing the points (a,b) and (c,d). If $a > c$ and $b < d$, then f is

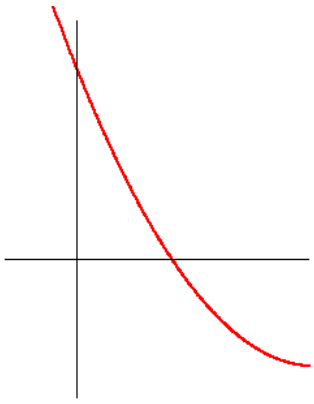
a. an increasing function.

b. a decreasing function.

c. a constant function.

d. a relation that is not a function.

18. Consider the graph of $f(x)$. If $f(0) = f^{-1}(a)$, what is the value of a?



19. If $f(x) = x^2$ and $g(x) = f(-2x)$, then which of the following is true?
    a. $g(x) = 4f(x)$
    b. $g(x) = -4x^2$
    c. $g(x) = -2f(x)$
    d. $g(x) = 4x$
    e. none of the above

20. Solve for x: $\log(3x - 1) = 2$.

21. If a chemical decays at a continuous rate of 3% per year, what is its half-life?
    a. $\dfrac{\ln(0.5)}{\ln(1.03)}$ b. $\dfrac{\ln(0.5)}{\ln(0.03)}$ c. $\dfrac{-\ln(2)}{\ln(0.03)}$ d. $-\dfrac{\ln(0.5)}{0.03}$

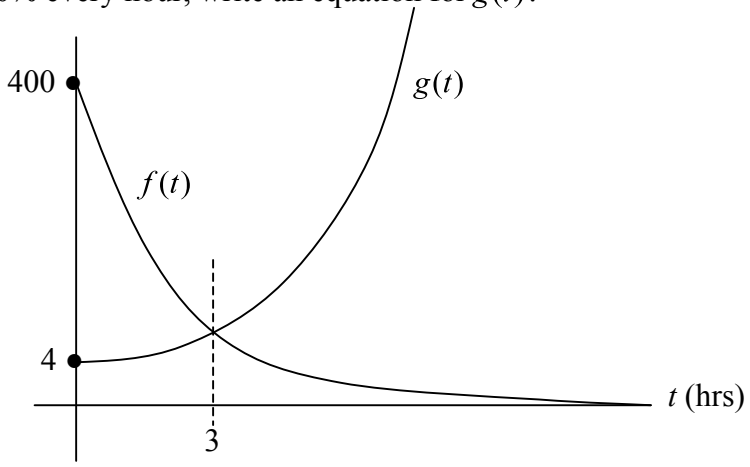
Answer all 6 questions in this section of part I. Each question is worth 3 points. Leave all answers in EXACT form, i.e., in terms of e, π, ln, $\sqrt{\ }$, etc., unless otherwise instructed. On this section, partial credit will be given. Therefore, show all work on the answer sheet in the space provided.

22. Three lines intersect at the same point. Two of the lines have the equations

$$y = \frac{4}{3}x \text{ and } y = 2x - 6.$$

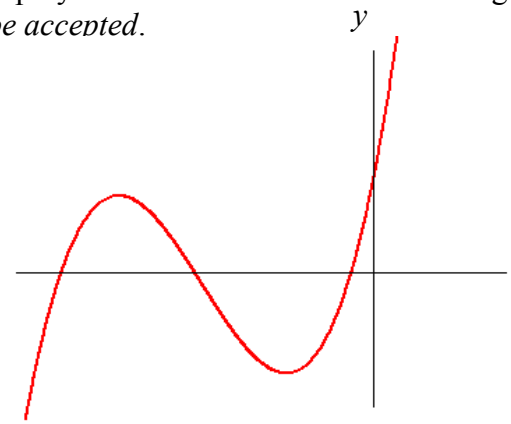
    The third line has a slope of – 2. Where does the third line intersect the y-axis?
    Only an algebraic solution will be accepted.

23. The graphs of exponential functions $f(t)$ and $g(t)$ intersect at $t = 3$, where t is in hours. If $f(t)$ decreases by 50% every hour, write an equation for $g(t)$.



24. Given the function $\ln y = x + \ln(x + 1) + \ln c$, express y as a function of x in a form that does not contain natural logs. Assume that c is a positive constant.

25. Find a formula for the polynomial function of *smallest* degree shown below. *Only an algebraic solution will be accepted*.



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26. The number of insects in a population is given by $P = P_0 e^{kt}$, where t is in months. If the population is initially 3000 insects and it doubles in 5.78 months, how long would it take for this population to triple? Round your answer to two decimal places.

27. A clock hangs on a wall so that its center is 50 inches above the floor. The minute hand of the clock is 4 inches long. Using a function of the form $H(t) = A\cos(Bt) + C$, write a function to describe the height of the tip of the minute hand above the floor, H, in inches, as a function of the time, t, in minutes, from midnight.

Part II: Before you begin, spend a few minutes reading each question. Answer *only* 5 *questions* in this part. Each question is worth 10 points. Be sure you clearly indicate the questions you *do not* wish to be graded. Partial credit will be given. Therefore, show all work in the blue book.

1. George and Laura had been waiting in line a long time to get into a theater. George became impatient and counted the number of people in front of them every 5 minutes, starting at 8:00pm. **(10)**

Time	8:00pm	8:05pm	8:10pm	8:15pm
No. of People (N)	245	218	198	146

George decided that he could model the number of people ahead of them, N, as a linear function of time, t, where t is the number of minutes after 8:00pm.

a. Write the function $N(t)$. Round any constants to one decimal place.

b. According to George's model, how many people would be in front of the couple at 8:30pm? (Round your answer to the nearest integer)

c. Using this model, when were 200 people ahead of George and Laura?

d. Using this model, what time could George and Laura expect to enter the theater?

e. In actuality, they got into the theater at 9:00. Does this time agree with your answer to part (d)? If so, explain why. If not, why not?

2. In one of the original Star Trek episodes, the starship Enterprise was infested with little furry creatures called "tribbles". These creatures reproduced at an alarming rate. Dr. McCoy estimated that their population was doubling every day. There were four tribbles initially.
(1, 1, 2, 3, 3)

   a. Write a formula of the form $P(t) = P_0 b^t$ that gives the tribble population as a function of time, where t is measured in days.

   b. If left unchecked, how many tribbles would be on-board in 14 days?

   c. Each tribble took up 0.25 ft^3. The Enterprise contained 7.8×10^6 ft^3. About how long before the tribble population could no longer fit on the ship?

   d. A tribble predator population grows according to the formula $E(t) = E_0 b^t$. *Ten days after the initial infestation*, four predators began being bred in a lab on-board the Enterprise. Five days later, the population had grown to 972 predators. Find a formula for $E(t)$.

   e. It is known that each predator can consume 100 tribbles per day. Captain Kirk wants to introduce all the predators at one time and have them eradicate all the tribbles in one day. Estimate, to the nearest day, the time it took to breed enough predators to eradicate the entire tribble population in one day.

3. a. When a seated person breathes normally, the number of liters of air in his lungs t seconds after exhaling completely is given by $V(t) = d - a\cos\left(\frac{nt}{b}\right)$, where a, b, d, and n are positive constants. **(1, 1, 1, 2)**

      i. What is the maximum amount of air in the lungs?

      ii. What is the minimum amount of air in the lungs?

      iii. If the period of breathing is defined as the time between inhalations, what is the period of breathing?

      iv. How many breaths are taken per minute?

   b. Temperatures in Town A oscillate sinusoidally between 30°F at 4am and 60°F at 4pm. Write the following formulas: **(2, 2, 1)**

      i. The temperature in Town A in terms of t, where t is measured in hours from 4am.

      ii. The temperature in Town A in terms of t, where t is measured in hours from midnight.

      iii. The temperature in Town B in terms of t, where the temperatures are consistently 10°F cooler than in Town A and t is measured in hours from 4am.

4. a. Find a quadratic function in the form $f(x) = ax^2 + bx + c$ that satisfies each of the following conditions: **(5)**

- (1, 1) is the vertex of the graph of $f(x)$
- The y-intercept of the graph is (0, 6)

b. Consider the graph of a function $f(x)$. **(5)**

1. Suppose $t(x) = 3f(x) + 1$. If the point $(-2, 3)$ lies on the graph of $f(x)$, what is the corresponding point on the graph of $t(x)$?
2. Suppose $s(x) = f\left(\frac{1}{3}x\right) - 4$. If the point $(3, 3)$ lies on the graph of $f(x)$, what is the corresponding point on the graph of $s(x)$?
3. Suppose $w(x) = f(x + 3) - 5$. If the point $(0, 2)$ lies on the graph of $f(x)$, what is the corresponding point on the graph of $w(x)$?
4. Suppose $n(x) = f(x - 2) + 2$. If the point $(-4, -1)$ lies on the graph of $f(x)$, what is the corresponding point on the graph of $n(x)$?
5. Suppose $h(x) = -f(2x)$. If the point $(6, -2)$ lies on the graph of $f(x)$, what is the corresponding point on the graph of $h(x)$?

5. Mary and Peter compete in a hot dog eating contest in which the first one to finish their hot dogs wins. Because Peter is larger than Mary, he is given more hot dogs to eat. Peter starts off with 75 hot dogs and Mary starts off with 60. Peter eats at the rate of 4 hot dogs per minute and Mary eats at the rate of 3 hot dogs per minute. **(3, 4, 3)**

a. How many minutes will it take for them to have the same number of hot dogs left to eat?

b. To the nearest whole number, how many hot dogs will the loser still have uneaten when the winner wins?

c. The winner of the contest jumped up in celebration, only to fall and be scraped. The normal healing of wounds can be modeled by an exponential function. If A_0 represents the original area of the wound and if A equals the area of the wound after n days, then the formula $A = A_0 e^{-0.35n}$ describes the area of a wound on the n^{th} day following an injury. How long before the wound is 20% of its original size? Round your answer to the nearest tenth.

6. A moving company charges \$300 plus \$5.00 per mile traveled in the move. **(2, 2, 4, 2)**
   a. Write the average cost per mile, A, as a function of d, the number of miles moved.
   b. Use a sentence or two to explain in *economic terms*, why the graph of A has the long range behavior that it does.
   c. Find a formula for A^{-1} and explain its economic significance.
   d. The moving company is acceptable to a client if the average cost to move is less than \$8.00 per mile. Find the minimum number of miles that must be driven to achieve this goal.

7. Suppose $f(x) = x^2 - k$, $g(x) = x + b$ and $h(x) = x^2 + c$, where k, b, and c are positive integers, and $k \neq b^2$. Find the correct function shown in choices (*i*.) through (*vi*.) that fits the function description in each question (*a*.) through (*e*.) **(10)**
   a. no zeroes, one vertical asymptote, one horizontal asymptote
   b. no zeroes, two vertical asymptotes, one horizontal asymptote
   c. two zeroes, one vertical asymptotes, no horizontal asymptotes
   d. one zero, no vertical asymptotes, one horizontal asymptote
   e. two zeroes, no vertical asymptotes, one horizontal asymptote

i. $j(x) = \dfrac{f(x)}{g(x)}$ *ii*. $s(x) = \dfrac{g(x)}{f(x)}$ *iii*. $m(x) = \dfrac{1}{g(x)}$

iv. $n(x) = \dfrac{g(x)}{h(x)}$ *v*. $r(x) = \dfrac{h(x)}{f(x)}$ *vi*. $w(x) = \dfrac{f(x)}{h(x)}$