NCC Pre Calculus Partnership Program
Final Examination, 2008

Part I: Answer all 25 questions in this part. Each question is worth 2 points. Leave all answers in EXACT form, i.e., in terms of $e, \pi, \ln, \sqrt{\ }$, etc., unless otherwise instructed. No partial credit will be given.

1. Among the tables below, which expresses y as a function of x?

(T1)

x	2	1	2	1
y	1	2	3	4

(T2)

x	0	π	π^2	π^3
y	0	1	0	-1

(T3)

x	-2	-1	0	1
y	-2	-1	0	1

(T4)

x	1	e^0	e^2	e^3
y	0	1	2	3

   a. All of them
   b. Only (T2), (T3), and (T4)
   c. Only (T3) and (T4)
   d. Only (T2) and (T3)
   e. Only (T3)

2. Let $f(x) = 4 - x^2$. Find the average rate of change of $f(x)$ on the interval $b \le x \le 2b$.

3. The expression $\dfrac{10^{\log 4x} + e^{\ln 6x}}{\arctan(\tan 10y)}$ is equivalent to

   a. $\dfrac{x}{y}$ b. $\dfrac{24x^2}{10y}$ c. $\dfrac{24}{10}$ d. $\dfrac{10e^{10x}}{10y}$ e. Can't be determined

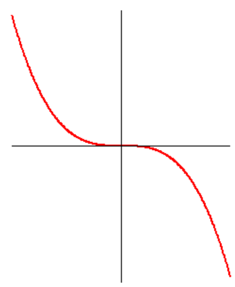
4. Jared runs cross country. At the beginning of a race, he has a lot of energy. As the race begins, he loses energy slowly, but as the race continues, he loses energy more and more rapidly. Jared's energy is
   a a decreasing linear function
   b. an increasing function whose graph is concave up
   c. a decreasing function whose graph is concave up
   d. an increasing function whose graph is concave down
   e. a decreasing function whose graph is concave down

5. The graph of a function $g(x)$ is horizontally stretched, then flipped over the x-axis and finally shifted up k units to produce a new function $f(x)$. Which of the following could be a formula for $f(x)$?
   a. $f(x) = -2g(x) + k$
   b. $f(x) = -g(0.5x) + k$
   c. $f(x) = -g(2x) + k$
   d. $f(x) = 0.5g(-x) + k$

6. If $f(t)=20\sin(0.3t)+30$, what is the range of $g(t)=f(t)-50$?

7. The volume of a sphere with radius r is given by the formula $V=f(r)=\frac{4}{3}\pi r^3$. Which expression represents the volume of a sphere whose radius has been increased by 6%?
   a. $0.06f(r)$
   b. $f(1.06r)$
   c. $f(r+0.06)$
   d. $f(0.06r)$
   e. $f(r)+0.06$

8. The figure below shows a mystery *power* function, $g(x)$. Which of the following statements must be true?



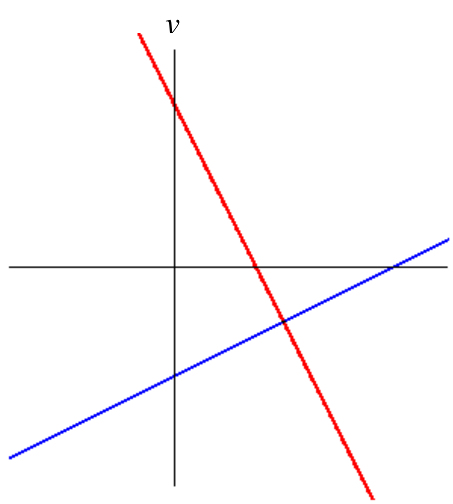
i. If the point $(-1, 3)$ lies on the graph of $g(x)$, then so does the point $(1,-3)$.

ii. The graph of $g(x)$ passes through the point $(0, 0)$.

iii. The function $g(x)$ is invertible.

a. Statements (*i*) and (*iii*) only. b. Statements (*ii*) and (*iii*) only.
c. Statements (*i*) and (*ii*) only. d. All statements are true

9. Consider the perpendicular lines shown in the figure below. If the slope of one line is -2, find the ***exact coordinates*** of the point of intersection of the two lines.



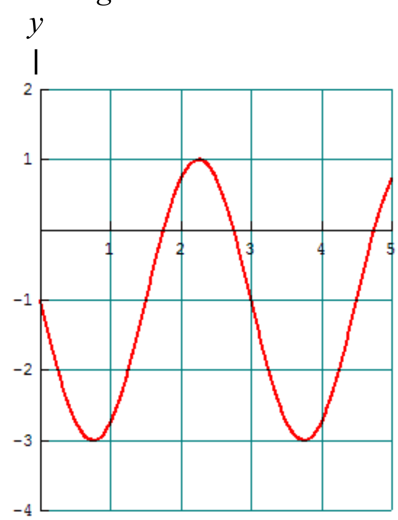
10. Given the function $f(x) = \dfrac{16}{x^2 - 9} + 2,$ which of the following is true about $f(x)$?

   a. It is neither odd nor even
   b. It is a quadratic function
   c. $-f(x) = f(-x)$
   d. $f(x) = f(-x)$

11. The expression $-c\ln(b) + \ln(a)$ is equivalent to

   a. $-c\ln(ab)$
   b. $-c\ln(a+b)$
   c. $\ln\left(\frac{a}{b^c}\right)$
   d. $e^{\left(\ln ab^{-c}\right)}$
   e. $\dfrac{a}{b^c}$

12. Find a possible formula for the *trigonometric* function whose graph is shown below.

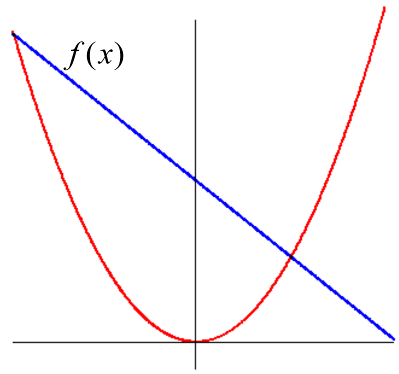


13. Let $p = f(h),$ where p is the pressure, in pounds per square inch, at an elevation of h thousand feet above sea level. What is the meaning of $f^{-1}(20)$?

   a. The elevation when the pressure is 20,000 pounds per square inch.
   b. The pressure when the elevation is 20 feet above sea level.
   c. The elevation when the pressure is 20 pounds per square inch.
   d. The pressure when the elevation is 20,000 feet above sea level.

14. In 2003, the population of city was 25,000 people. Over a four year period, the population grew by 20 percent. Find a formula, in the form $P = P_0 e^{rt}$, for the population of the city as a function of t, the number of years since 2003. Round any constants to 4 decimal places.

15. Find a formula for a linear function, $f(x)$, whose graph is shown in the figure below.

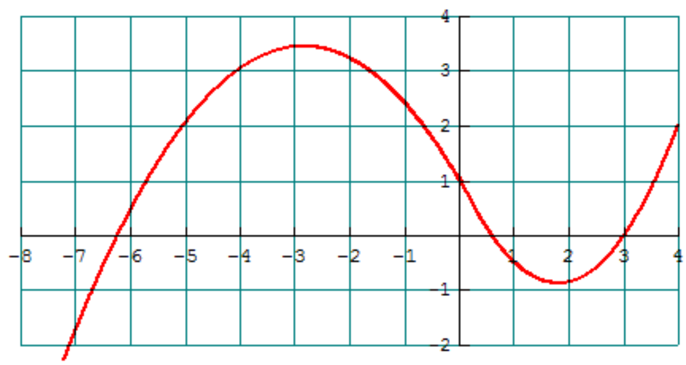


For questions 16 – 19: consider the four functions f, g, h, and u shown below.

x	-4	-3	-1	3
$g(x)$	4	-4	-0.5	0.3

$$h(x) = \frac{3 - \sqrt{x}}{\sqrt{x} + 2}$$

$$u(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$



16. What is the domain of $h(x)$?

17. What is the value of $g(f(-4))$?

18. What is the value of $h^{-1}(0)$?

19. Find the solution to $g^{-1}(x) = u(x)$.

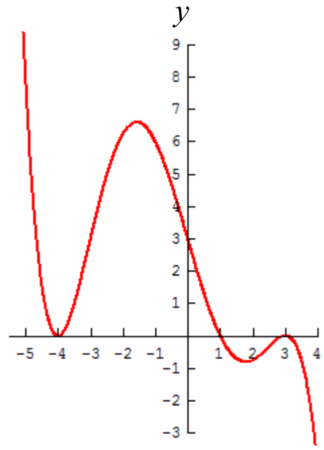
20. If $f(x) = \log(x)$ which of the following statements is *false*?

   a. The graph of $f(x)$ has a horizontal intercept at (1, 0).
   b. The graph of $f^{-1}(x)$ is concave up.
   c. As $x \to \infty$, $f(x) \to \infty$.
   d. The domain of $f(x)$ is $x \geq 0$.
   e. All the statements are true.

21. The inverse sine and inverse cosine functions have

    a. the same domain
    b. the same range
    c. both (a) and (b)
    d. neither (a) nor (b)

22. Find a possible formula for the function shown below.



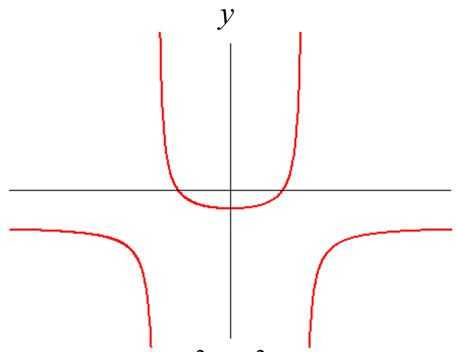
23. Consider the function $h(x) = a(x-b)^2(x-c)^m$. Assume the constant m is a positive integer and $a \neq 0$. Which of the following claims about $h(x)$ is true?

    i. The degree of $h(x)$ is $2m$.
    ii. The graph of the function $h(x)$ could pass through the point (0, 0).
    iii. The graph of $h(x)$ has exactly one y-intercept.

    a. All statements are true.
    b. Only statements *ii* and *iii* are true.
    c. Only statements *i* and *iii* are true.
    d. Only statement *iii* is true.

24. A student traveled to Europe for the summer and charged the trip on her MasterCard. Her credit card company charges a nominal rate of 15.25% compounded monthly. After she paid the minimum payment due, her balance was $2500. How much interest accrued on her account after 1 *month,* assuming she made no other purchases?

25. Consider the graph shown below. Which of the following functions could produce the graph below?



a. $y = \dfrac{1}{x^2 - a^2}$ b. $y = \dfrac{x^2 - a^2}{a - x^2}$ c. $y = \dfrac{a - x^2}{x^2 - a^2}$ d. $y = \dfrac{x^2 - a}{x^2 - a^2}$

Part II: Before you begin, spend a few minutes reading each question. ***Answer only* 5 *questions in this part***. Each question is worth 10 points. Be sure you clearly indicate the questions you *do not* wish to be graded. Partial credit will be awarded for work that is partially correct. Therefore, show all work in the blue book.

1. Andrea is bouncing on a trampoline. The trampoline surface is 1 meter above the gym floor. When Andrea bounces up, her feet reach a maximum height of 3 meters above the surface. When she bounces down, her feet depress the surface at most 0.5 meters. The coach allows Andrea a few practice bounces, and then starts to time her when she is at her highest point. The coach noted that Andrea was at her lowest point at 1 second. **(2, 3, 3, 2)**
   a. Sketch one complete cycle of the graph which shows the position of Andrea's feet above the gym floor, H, as a function of time, t. Be sure to label all pertinent information.
   b. Determine a sinusoidal equation that represents the position of Andrea's feet above the ground as a function of time.
   c. During the first cycle, when were Andrea's feet 3 meters above the floor? Round your answer to the nearest hundredth of a second. *Only an algebraic solution* will be given full credit.
   d. The coach decided to time Andrea again. This time he noted that at 0.5 seconds, Andrea is at her maximum height (3 meters) and at 1.5 seconds, she is at her minimum height (0.5 meters.) Determine a sinusoidal equation that represents the position of Andrea's feet above the ground as a function of time.

2. At the University of Arizona, the spring semester is marked by a student-organized fair called "Spring Fling". This week-long event features, among other outdoor activities, music, food, and roller-coaster rides. Let R be the amount of money collected on the first day of Spring Fling. Assume that R is a function of the total number of hours, A, that visitors spend collectively on roller-coaster rides on that day. Interpret the meaning of each of the following expressions in the context of the Arizona's Spring Fling scenario. **(10)**

   a. $R(A)$ b. $R(A) - 100$ c. $R(A - 100)$ d. $100R(A)$ e. $R^{-1}(100)$

3. The average life-span of a mystical bird species, called nybors, has increased over time. (So, birds born many years ago lived, on average, a shorter time than birds born more recently.) The nybors inhabit a mystical forest where trees have begun to turn blue. Researchers started observing the nybors 3 years after trees began to turn blue and found that the birds lived an average of 7 years. Ten years later, it was found that the average life-span of the nybors had increased to 15 years. **(3, 4, 1, 2)**

   a. If life span is a linear function of time, find a formula giving the average life-span of the nybors, L, measured in years, as a function of time, t, measured in years since the trees began to turn blue.

   b. If life span is an exponential function of time, find a formula giving the average life-span of the nybors, L, measured in years, as a function of time, t, measured in years since the trees began to turn blue. Assume *continuous growth* and round any constant to 4 decimal places.

   c. Suppose that the average life-span of nybors is given by $L = f(t) = \dfrac{-150}{t+3} + 53$, where again, L is measured in years and t is measure in years since the trees in the mystical forest began to turn blue. How many years did the nybors live on average when the trees began to turn blue?

   d. Using the equation in part (c), what will be the average life-span of the nybors over many, many years? Be specific. Write a sentence or two explaining how you arrived at your answer.

4. The police discovered the body of a murder victim. Critical to solving the crime is determining the time at which the murder was committed. The coroner arrived at the murder scene at 12:00 p.m. She immediately took the temperature of the body and found it to be 94.6°. One hour later, she took the temperature again and found it was 93.4°. The room in which the murder had been committed maintained a constant temperature of 70°. It is known that when an object with an initial temperature, H_0, is placed in surroundings in which the temperature is kept constant, H_c, the temperature of the object will cool over time according to the formula $H(t) = H_c + (H_0 - H_c)e^{kt}$. **(2, 4, 4)**

   a. Rewrite the formula $H(t) = H_c + (H_0 - H_c)e^{kt}$ so it only contains the variable t and the parameter k.

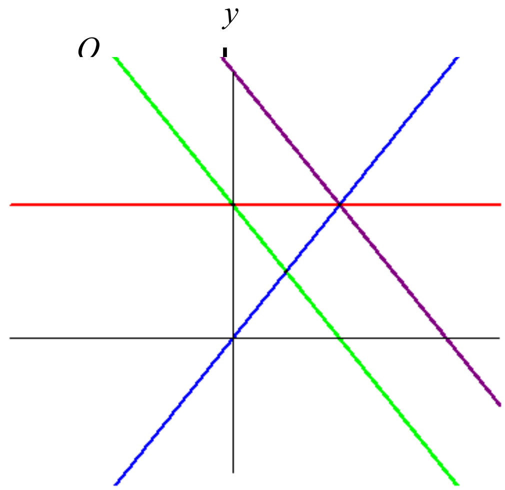
   b. Use the equation you obtained in part (a) to solve for k. Round your answer to 2 decimal places.

   c. To the nearest hour, when was the murder committed? (Normal body temperature is approximately 98.6°.)

5. a. Solve $2\sin^2(\theta)\cot(\theta) = 0.5\cot(\theta)$ for x on the interval $[0, 2\pi)$. *Only an algebraic solution with exact values* will be given full credit. All work must be shown to receive partial credit. Calculator solutions will receive no credit. **(5)**

   b. Let $f(x) = 10e^{\frac{x-1}{2}}$ and $g(x) = 2\ln x - 2\ln 10 + 1$. Show that $f(g(x)) = x$, for all x. **(5)**

6. The graph shown below depicts four linear functions, labeled P, Q, R, and S. The functions y_1, y_2, y_3, and y_4 are the equations of the four lines, and the constants a, b, and c are all positive. **(4, 2, 1, 3)**



   a. Each equation below matches one and only one of the above lines. Write the name of the line to which the equation belongs.

   i. $y_1 = \dfrac{x}{b}$ *ii.* $y_2 = a$ *iii.* $y_3 = c - bx$ *iv.* $y_4 = a - bx$

   b. Find the x-intercept of line Q.

   c. Which line depicts the graph of a direct proportion?

   d. In terms of a, b, and c determine the *coordinates* of the point where the line Q intersects the line S.

7. a. Suppose $N(p) = -\dfrac{1}{10}(p-65)^2 + 360$ represents the profit (in dollars) that a vendor earns each day by selling coffee for p cents per cup. **(3, 1, 1)**

   i. On what price interval is the vendor guaranteed to sell his coffee at a profit? (***Only an algebraic solution will be accepted***)

   ii. What price should the vendor charge for a cup of coffee in order to earn the maximum profit per day?

   iii. What is this maximum profit?

   b. A fraternity house plans to section off a portion of their brick patio to build a spring flower garden. The sides of the garden not bordering the brick patio will be surrounded by a fence. There are only 88 yards of fence available. With the limitations imposed by the available fence, the area A of the garden can be written as a function of its width w by:

$$A(w) = w(-2w + 88) = -2w^2 + 88w.$$

   i. Put the equation of $A(w)$ in vertex form by *completing the square*. All work must be shown.

   ii. Find the dimensions w and l of the garden that make its area as large as possible. **(4, 1)**