2009 Final Exam
Answers, Part I

1. $1.6\ \frac{\text{dollars}}{\text{pound}}$ or $\frac{8}{5}\frac{\text{dollars}}{\text{pound}}$

2. 2

3. E

4. $\dfrac{\text{miles}/\text{gallon}}{\text{miles}/\text{hour}}$ or $\dfrac{\text{hours}}{\text{gallon}}$

5. 7.33%

6. E

7. $\dfrac{\sqrt{2}}{2}$

8. C

9. D

10. $x > -\dfrac{c}{k}$ or $\left(-\dfrac{c}{k}, \infty\right)$

11. 5518

12. -1.4

13. 37 or 37,000

14. D

15. b + c

16. 3

17. D

18. $p(x) = \frac{2}{3}x(x-2)(x+2)^2$

19. b

20. $(-\infty, -4]$ or $y \le -4$

21 $c + k$

22. $k = 4$

23. $y = \dfrac{2}{\log 2 - 1}$ or $\dfrac{-2}{1 - \log 2}$

24. A

25. $(0.9)^4 l_0$ or $0.6561 l_0$

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Solutions, Part II

1. a. *i.* The slope will be $m = -4$ so the line will be $y - 12 = -4(x-3)$ or $y = -4x + 24$.

   ii. The points $(2, 3.65)$ and $(10, 6.45)$ are on the line, Its slope is $m = \frac{2.8}{8} = 0.35.$

   Therefore, $C - 3.65 = 0.35(n-2)$ or $C = 0.35n + 2.95.$

   iii. The slope is $m = \frac{k-0}{c-0} = \frac{k}{c}.$ Therefore $f(x) = \frac{k}{c}x.$

   b. *i.* We have $10I + 15K = 60$, so $K = 4 - \frac{2}{3}I.$

   ii. The horizontal intercept, 6, represents the number of pounds of Italian coffee one can purchase if no Kenyan coffee is purchased.

   iii. The slope tells us that for every 3 lbs. of Italian coffee bought, 2 lbs. less of Kenyan coffee can be bought.

2. a. *i.* $q = 1200 - 3p = 0 \rightarrow p = \$400/\text{chair}.$

   ii. $R = p(1200 - 3p) = -3p^2 + 1200p$

   $= -3(p^2 - 400p) = -3(p^2 - 400p + 40{,}000) + 120{,}000$

   $= -3(p - 200)^2 + 120{,}000.$

   Therefore, $200/chair.

   b. *i.* $r = 200t.$ *ii.* $A = \pi r^2 = \pi(200t)^2 = 40{,}000\pi t^2.$

   iii. $6{,}157{,}521.601 = 40{,}000\pi t^2 \rightarrow t = \sqrt{\frac{6{,}157{,}521.601}{40{,}000\pi}} \approx 7$ hours.

3. a. *i.* $M(t) = M_0 e^{-0.002t}.$

   ii. $V(t) = V_0\left(1 + \frac{0.02}{12}\right)^{12t}.$

   b. *i.* $-40^\circ F$; this is the initial temperature of the sample.

   ii. After a long period $(t \rightarrow \infty)$ $H(t) \rightarrow 30;$ this corresponds to the horizontal asymptote.

   iii. $20 = 70\left(1 - 2^{-0.05t}\right) - 40$

   $2^{-0.05t} = \frac{1}{7}$

   $t = -\frac{\log(1/7)}{0.05\log 2}$ or $\frac{\log(1/7)}{0.05(\log 0.5)} \approx$ 56 minutes.

4. a. \$15 b. \$190.40 (\$206 – 15.60 discount)

 c. *i.* \$15

 ii. It is the original price of an order whose final price is \$15

 d. $$f(x) = \begin{cases} x & \text{if } 0 \le x < 70 \\ x - 15 & \text{if } 70 \le x \le 200 \\ x - [15 + 0.10(x - 200)] & \text{if } x > 200 \end{cases}$$

5. a. $2\cos^2 t = 3\sin t + 3$

$$2\left(1 - \sin^2 t\right) = 3\sin t + 3$$
$$2\sin^2 t + 3\sin t + 1 = 0$$
$$(2\sin t + 1)(\sin t + 1) = 0$$
$$\sin t = -\frac{1}{2} \text{ or } \sin t = -1$$
$$t = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}.$$

 b. $e(5^{3x}) = e^x$

$$5^{3x} = e^{x-1}$$
$$3x\ln 5 = x - 1$$
$$3x\ln 5 - x = -1$$
$$x = -\frac{1}{3\ln 5 - 1} = \frac{1}{1 - 3\ln 5}.$$

OR

$$e(5^{3x}) = e^x$$
$$e = \frac{e^x}{5^{3x}} = \left(\frac{e}{5^3}\right)^x$$
$$1 = x\ln\left(\frac{e}{5^3}\right).$$
$$x = \frac{1}{\ln\left(\frac{e}{5^3}\right)} = \frac{1}{1 - 3\ln 5}.$$

6. a. *i.* $y = -5\cos\left(\frac{\pi}{2}t\right) + 6.$ or the equivalent

 ii. On average, the ranking is 6.

 b. *i.* $y = 6\cos\left(\frac{2\pi}{3}t\right) + 10.$

 ii. $8 = 6\cos\left(\frac{2\pi}{3}t\right) + 10$

$$-\frac{1}{3} = \cos\left(\frac{2\pi}{3}t\right)$$
$$t = \frac{3}{2\pi}\arccos\left(-\frac{1}{3}\right)$$
$$t_1 = \frac{3}{2\pi}\arccos\left(-\frac{1}{3}\right) \qquad t_2 = 3 - \frac{3}{2\pi}\arccos\left(-\frac{1}{3}\right).$$

7. a. $k = 2, \quad p = -4, \quad q = 6, \quad r = -2, \quad s = 3.$

 b. *i.* The graph of $f(x)$ concave up. The rate of change is increasing.

 ii. $b - (b-3) = 3b - (b-3)$

 $2b = 6 \rightarrow b = 3.$

 iii. If $b = 0$ then $g(0) = 0$ and $g(-2) = 0.$

 Therefore, $g(x)$ would not have an inverse.