

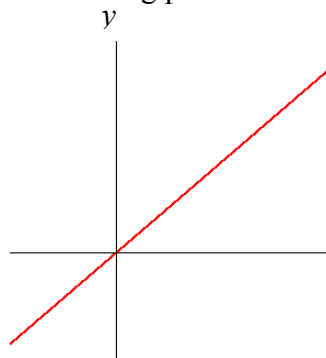
**NCC Pre Calculus Partnership Program
Final Examination, 2009**

Part I: Answer all 25 questions in this part. Each question is worth 2 points. Leave all answers in EXACT form, i.e., in terms of e , π , \ln , $\sqrt{\quad}$, etc., unless otherwise instructed. No partial credit will be given.

1. When you convert British pounds (£) into US dollars (\$), the number of dollars you receive is directly proportional to the number of pounds you exchange. A traveler receives \$400 in exchange for £250. Find the constant of proportionality, *including units*.
2. Find the period of the graph of $y = c \sin(\pi(x - k)) + d$. Assume c , d and k are constants.
3. On February 28, 2009, an article in *The New York Times* stated “The economy is spiraling down at an accelerating pace; threatening to undermine the Obama administration’s spending plans” Based on this statement, which of the following best describes a function that represents the economy?
 - a. It is a decreasing linear function.
 - b. It is an increasing function whose graph is concave up.
 - c. It is a decreasing function whose graph is concave up.
 - d. It is an increasing function whose graph is concave down.
 - e. It is a decreasing function whose graph is concave down
4. An airplane’s fuel consumption is given by $g = f(v)$ where v is measured in miles per hour and g is measured in miles per gallon. What are the units of the average rate of change of the graph g ?
5. Radio-isotopes of different elements have different half-lives. Magnesium-27 has a half-life of 9.45 minutes. What is the continuous percent decay rate for Magnesium-27? (*Your answer must be correct to the nearest hundredth of a percent.*)
6. If $a < b$ and the average rate of change of $y = f(x)$ between $x = a$ and $x = b$ is -3 , then which of the following *must be true*?
 - a. The graph of $f(x)$ is concave up
 - b. The graph of $f(x)$ is concave down
 - c. $f(x)$ is linear
 - d. $f(x)$ is exponential
 - e. $f(a) > f(b)$
7. Suppose $m(x) = \sin(x)$, $p(x) = x - \frac{\pi}{4}$ and $q(x) = m(p(x))$. Find the exact value $q\left(\frac{\pi}{2}\right)$.
8. If the $\sin \theta = m$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ then θ is

a. $\frac{1}{m}$	b. $\frac{1}{\sin m}$	c. $\arcsin m$	d. $\arcsin \theta$	e. $\frac{m}{\sin}$
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9. Consider the graph shown below. If a new line, L , (*not shown*) is drawn from the origin and intersects the line segment connecting points a and b , then a possible slope for line L is



- a. 2 b. $\frac{5}{8}$ c. $\frac{1}{2}$ d. $\frac{1}{4}$ e. $\sqrt{2}$
10. What is the domain of $f(x)$ if $f(x) = 3 \log(kx + c)$? Assume the constants c and k are both positive.
11. A town's population is initially 15,000 and is decreasing at a continuous rate of 5% per year. What is the population of this town 20 years later? *Round your answer to the nearest whole number.*
12. Consider the table shown below. What is the value of p if $f(x)$ is a linear function?

x	1.1	3.2	5.6
$f(x)$	7.6	3.4	p

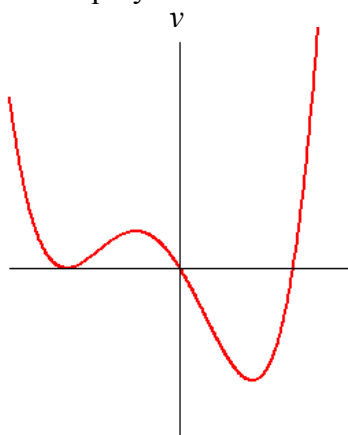
13. Average walking speed in a city is a function of the size of its population, since as population increases, so does the pace of life. Average walking speeds can be modeled by the function $w = f(p) = 0.35 \ln p + 2.74$, where p is the population, *in thousands*, and w is the average walking speed in feet per second. Solve $f(p) = 4$. *Round your answer to the nearest thousand.*
14. The graph of $f(x) = x^2$ is shifted horizontally, resulting in the graph of $g(x)$. Which of the following tables displays some of the points on the graph of $g(x)$? (Assume the number of units shifted is not 0).

- a. $\begin{array}{c|cccc} x & -1 & 0 & 1 & 2 \\ \hline y & 1 & 0 & 1 & 4 \end{array}$ b. $\begin{array}{c|cccc} x & -1 & 0 & 1 & 2 \\ \hline y & -1 & 0 & -1 & -4 \end{array}$ c. $\begin{array}{c|cccc} x & -1 & 0 & 1 & 2 \\ \hline y & 2 & 1 & 2 & 5 \end{array}$ d. $\begin{array}{c|cccc} x & -1 & 0 & 1 & 2 \\ \hline y & 0 & 1 & 4 & 9 \end{array}$

15. Find $\lim_{t \rightarrow \infty} b(1 - e^{-0.8t}) + c$ if b and c are constants.

16. The point $(0,1)$ is contained on the graph of the function $f(x)$. If $g(x) = -2f(4(x-2)) + 5$, and the point $(2, k)$ is on the graph of $g(x)$, what is the value of k ?
17. If $n = f(p)$ gives the average number of days a house stays on the market before being sold for a price p (in \$1000s), and p_0 is the average selling price of houses in a particular community, then $f(0.9p_0)$ represents
- the average selling price of a house if the average selling price is reduced by 90%.
 - the average number of days a house stays on the market if the average selling price is reduced by 90%.
 - the average selling price of a house if the average selling price is reduced by 10%.
 - the average number of days a house stays on the market if the average selling price is reduced by 10%.
 - none of these.

18. Find a possible formula for the polynomial function $p(x)$, whose graph is shown below.



19. The table shown below displays some ordered pairs of the functions f and g . If f is an even function, g is an odd function and h is the composition $h(x) = f(g(x))$, find $h(a)$.

x	$-a$	$-b$	0	b	a
$f(x)$	a	b	c		
$g(x)$	$-b$	a	0		

20. Suppose $f(x) = (x+m)^2 + 4$ and $g(x) = -f(x)$. What is the range of g ?

21. Suppose the sales, S , in hundreds of units, of a seasonal product is modeled by $S = c - k \cos\left(\frac{\pi t}{6}\right)$ where t is measured in months and c and k are positive constants. What is the maximum number of units sold?
22. Find k such that the function $f(t)$, represented in the table below, could represent an exponential function. All output values for f should be positive.

t	3	4	5
$f(t)$	k	$k + 4$	$5k - 4$

23. Find the *exact* solution to the equation $y + 2 = \log(2^y)$
24. The expression $\log 100a - \log a + e^{\ln a + \ln b}$ is equivalent to which of the following expressions?
- $2 + ab$
 - $100 + a + b$
 - $100 + ab$
 - $2ab$
 - $2 + a + b$
25. A patient had an uncontrolled cholesterol level of l_0 , that was deemed too high. When given one pill per day of Lipitor to control the cholesterol, his cholesterol dropped to $0.9l_0$. After a few months the dose was increased to two of the same strength pills per day to further lower his cholesterol. This dose reduced his cholesterol to $0.81l_0$. Assuming all other factors affecting cholesterol level remained constant and assuming that the patient dose-response function is exponential, to what will the cholesterol be reduced if the dose is raised to four pills of the same strength per day? Your answer should be in terms of l_0 .

Part II: Before you begin, spend a few minutes reading each question. Answer *only 5 questions* in this part. Each question is worth 10 points. Be sure you clearly indicate the questions you *do not* wish to be graded. Partial credit will be awarded for work that is partially correct. Therefore, show all work in the blue book.

1. a. Find a formula for the linear functions described in parts *i-iii*. (3)
 - i.* The function $h(x)$, whose graph is parallel to the line $y - 8 = -4(x - 3)$ and contains the point $(3, 12)$.
 - ii.* The total cost C of an international call lasting n minutes if two minutes cost \$3.65 and ten minutes cost \$6.45
 - iii.* The function, $y = f(x)$, where $f(c) = k$ and the graph of $f(x)$ goes through the origin.
 - b. A gourmet coffee shop has a weekly budget for two imported coffee beans. Sixty dollars per week is allotted for Italian beans and Kenyan beans. Italian beans cost \$10/lb and Kenyan beans costs \$15/lb. (3, 2, 2)
 - i.* Write a formula for the number of pounds of Kenyan beans, K , the gourmet coffee shop can buy as a function of the number of pounds of Italian beans, I , it can buy.
 - ii.* If the function in part (*i*) is graphed, what does the horizontal intercept represent *in the context of this problem*?
 - iii.* Interpret the slope of the line *in the context of this problem*.
2. a. A carpenter finds that if he charges p dollars for a chair, he sells $1200 - 3p$ chairs each year. That is, $q = 1200 - 3p$, where q is the number of chairs sold. (1, 4)
 - i.* At what price will the carpenter price himself out of the market, that is, have no customers at all?
 - ii.* Revenue is defined to be the product of the number of items sold times the price per item, that is, $R = pq$. *By using the method of completing the square*, find the price per item that maximizes the revenue.
 - b. Two oil tankers crash in the Pacific Ocean, creating an oil spill. The spreading oil has a circular shape, and the radius of the circle is increasing at 200 meters per hour. (1, 2, 2)
 - i.* Express the radius of the spill, r , as a function of time t , in hours since the crash.
 - ii.* Express the area of the spill, A , as a function of time t .
 - iii.* When the EPA began the clean-up, the oil spill covered an area of 6,157,521.601 square meters. How many hours ago did the ships collide?

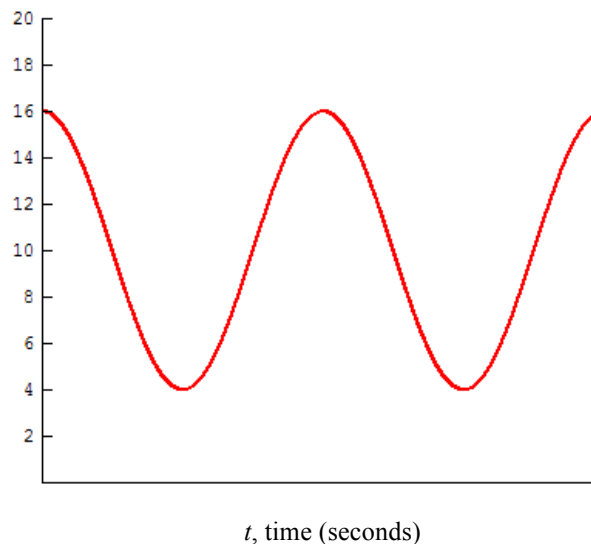
3. a. Find a *formula* for each functions described in parts *i-ii*. **(2)**
- A sample of a radioactive element, with an initial mass of M_0 grams, decays at a continuous of rate 0.2% per hour. Find $M(t)$, the number of grams that remain after t hours have elapsed.
 - An initial investment of V_0 dollars is compounded monthly at a nominal rate of 2% per year. Find $V(t)$, the value of the investment after t years.
- b. In order to study the effects of global warming, a group of meteorologists was dispatched to the South Pole. The temperature, in degrees Fahrenheit, of an ice sample taken from below the surface is given by $H(t) = 70(1 - 2^{-0.05t}) - 40$, where t is measured in minutes since the sample was removed from the ground. **(2, 2, 4)**
- What is the value of the vertical intercept of the graph of $H(t)$ and *what does it tell you about the ice core?*
 - What will happen to the temperature of the core after a long period of time? *To what feature of the graph of this function does this correspond?* Be specific!
 - The sample is best studied when its temperature is 20° F. Find the *exact* time when it should be studied, *then round your answer to the nearest minute*. (An algebraic solution is required, and partial credit will be awarded where appropriate. Solutions obtained using graphing calculators will not be accepted).
4. An online retailer is offering discounts to eligible customers. To be eligible for a discount, the customer must spend at least \$70. An eligible customer who spends \$200 or less will receive a discount of \$15. If a customer spends more than \$200, the discount will be \$15 on the first \$200 plus an additional 10% off the amount exceeding \$200. Assume there are no taxes on any of these transactions. **(1, 1, 2, 6)**
- What is the discount on an order of \$76?
 - What is the final price on an order of \$206?
 - Let $y = f(x)$ give the final price on an order of x dollars.
 - Evaluate $f^{-1}(15)$.
 - Interpret the meaning of $f^{-1}(15)$.
 - Express $y = f(x)$, the final price on an order of x dollars as a piecewise function defined for all $x \geq 0$.

- 5.a. Solve the equation $2\cos^2 t = 3\sin t + 3$ for t on the interval $[0, 2\pi)$. *Only an algebraic solution with exact values* will be given full credit. All work must be shown to receive any credit. Calculator solutions will receive no credit. Recall that $\sin^2 \theta + \cos^2 \theta = 1$ (5)
- b. Find the *exact* solution of the equation $e(5^{3x}) = e^x$. All work must be shown to receive partial credit. Calculator solutions will receive no credit. (5)
6. a. The popularity of the color green in women's fashion has fluctuated throughout the years. Its popularity was at a peak in 1989 as show in the following table. (A ranking of 1 is the highest). (3, 1)

t (time in years since 1989)	0	1	2	3	4	5	6	7	8
r (popularity rank)	1	6	11	6	1	6	11	6	1

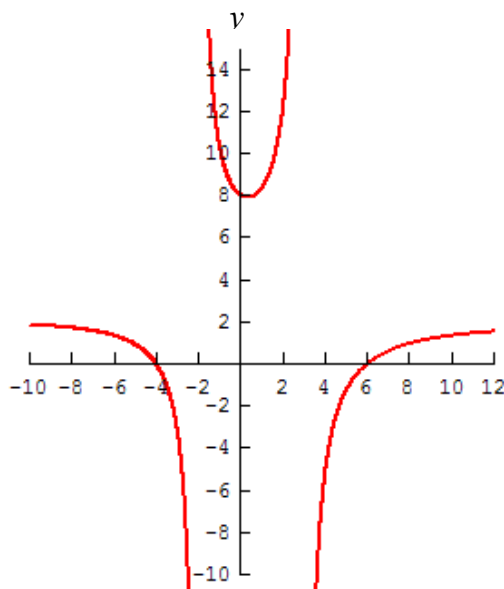
The data in the above table suggests that the popularity ranking, r , of green in women's fashion could be modeled by a sinusoidal function.

- Write a formula for the popularity ranking, r , as a function of t in years since 1989.
 - On average, how high does the color green, rank in popularity in women's fashion?
- b. In a baby's room, a toy is suspended from a bar that hangs over the baby's crib. When the baby pulls and releases the toy, it begins to bob up and down so that the distance between the toy and the bar oscillates in a sinusoidal fashion. Let $d(t)$ be the toy's distance from the overhanging bar, measured in centimeters, after t seconds. The graph of $d(t)$ is shown below. (2, 4)



- Find a possible formula for the sinusoidal function $d(t)$.
- Find all times in the first 3 seconds for which the toy is 8 centimeters from the bar. Algebraically derive your answers, show all appropriate work, and state your answers in *exact form*.

7. a. The graph of the rational function $y = \frac{k(x-p)(x-q)}{(x-r)(x-s)}$ is shown below. Based on the graph, what are the values of k , p , q , r , and s where $p < q$ and $r < s$? (5)



7. b. Two (unrelated) functions $y = f(x)$ and $z = g(x)$ are described using the table below.

x	-3	-2	-1	0
$y = f(x)$	11	13	16	20
$z = g(x)$	$b-3$	b	$b+3$	$3b$

Assuming that b is a constant, answer each of the following questions and give reasons for your answers. (1, 2, 2)

- Is it possible for the graph of $f(x)$ to be concave up or concave down? Explain your answer clearly using words and calculations as necessary.
- Is there a value of b such that $z = g(x)$ is linear? Explain your answer clearly.
- The first and third rows in the above table describe a relationship between x and $g(x)$. Find a value for the constant b such that $g(x)$ does not have an inverse. Explain why the value you found works.