2010 Final Exam
Answers, Part I

1. -5
2. C
3. B
4. $y = x + 6$ or equivalent
5. D
6. $y = -\frac{1}{3}(x+1)^2(x-3)$
7. $2a + h$
8. $C = 0.85(p-120) = 0.85p - 102$
9. B
10. $h - k$
11. $y = \dfrac{3(x+5)}{x-2}$
12. $x = 10$
13. B
14. $x = y^{3/2}$ or equivalent
15. C
16. B
17. A
18. E
19. B
20. C
21. D
22. A
23. 10
24. $(4, -2)$
25. $-\frac{1}{2}$

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Solutions, Part II

1. a. The data yields the two points $(218, 196.67)$ and $(382, 277.03)$. Thus, the slope is

$$m = \frac{277.03 - 196.67}{382 - 218} = \frac{80.36}{164} = 0.49.$$

This gives $y = 0.49x + b$. Using either point, $b = 89.85$. Hence, the equation is $y = 0.49x + 89.85$.

b. The slope is $\dfrac{\Delta y(\$)}{\Delta x(\text{miles})}$, so the units of the slope are cost per mile. Therefore, it costs 49 cents per mile.

c. The vertical intercept is the cost for renting a car for the 3 days independent of the mileage charge.

d. We have $160 = 0.49x + 89.85$. Thus, $x = \dfrac{160 - 89.85}{0.49} \approx 143.16$ miles. Since I do not want to exceed my budget, I must round down to 143.1 miles.

e. From part (a), we know the cost for the car for 3 days is \$89.85 with a constant daily rate. Thus, the cost per day is $\$89.85 \div 3 = \$29.95$. The mileage rate is \$0.49 per mile. Hence, the total cost will be $2 \times \$29.95 + 382 \times \$0.49 = \$247.08$.

2. a. $\text{Rev} = \left(-\frac{1}{6}q + 100\right) \cdot q = -\frac{1}{6}q^2 + 100q$

b. Either by using the axis of symmetry, $q = \dfrac{-b}{2a} = \dfrac{-100}{2 \cdot \left(-\frac{1}{6}\right)} = 300$ items

or by completing the square, we get $-\frac{1}{6}(q - 300)^2 + 15,000$ so maximum $q = 300$.

c. Maximum revenue, $R = \left(-\frac{1}{6} \cdot 300 + 100\right) \cdot 300 = (\$50)300 = \$15,000$

d. $R\,(q - 50)$ will have the same maximum revenue as the original function.

e. $R(q) + 100$ is a function where the quantity sold that produces the maximum revenue remains unchanged.

3. a Both curves intersect the y-axis at the same point. To find it, we find the equation of the line. The slope of the line is $m = \frac{4-0}{-2-1} = -\frac{4}{3}$. Using the point (1, 0) in the form $y = mx + b$ gives $b = \frac{4}{3}$, so the equation of the line is $y = -\frac{4}{3}x + \frac{4}{3}$. This means the line intersects the y-axis at $y = \frac{4}{3}$. Therefore, two points on $G(x)$ are $\left(0, \frac{4}{3}\right)$ and $(-2, 4)$, so $G(x) = \frac{4}{3}b^x$. Using the point $(-2,4) \to b = \sqrt{\frac{1}{3}}$. Hence, $G(x) = \frac{4}{3}\left(\sqrt{\frac{1}{3}}\right)^x$.

b. $P = P_0 b^t$

$2 = b^{40}$

$b = 2^{\frac{1}{40}}$

Therefore $P = 500\left(2^{\frac{1}{40}}\right)^t = 500\left(2^{\frac{t}{40}}\right)$

We need to solve

$$500(2)^{\frac{t}{40}} = 10,000$$

$$(2)^{\frac{t}{40}} = 20$$

$$\log(2)^{\frac{t}{40}} = \log 20$$

$$\frac{t}{40}\log 2 = \log 20$$

$$t = \frac{40\log 20}{\log 2} \approx 172.87$$

The guards should be commissioned about 172 minutes after the store opened, or about 10:52.

4. a. $f(t) = 4\sin(2\pi t) + 10$.

$12 = 4\sin(2\pi t) + 10$

$\frac{1}{2} = \sin(2\pi t)$

$2\pi t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \to \quad t = \frac{1}{12}, \frac{5}{12}, \frac{13}{12}, \frac{17}{12}$

b. Completing the square to find the vertex, we get:

$$y = (x-4)^2 + 5.$$

The vertex of the parabola is at $(4,5)$. Thus, from $x = 0$ to $x = 4$ the graph completes 1.5 cycles/periods. Therefore, the period (P) is
$P = \frac{4}{1.5} = \frac{8}{3}$, with $P = \frac{2\pi}{b} \Rightarrow \frac{8}{3} = \frac{2\pi}{b} \Rightarrow b = \frac{3\pi}{4}$. This gives $f(x) = A\cos\left(\frac{3\pi}{4}x\right) + c.$
The amplitude is $A = \frac{1}{2}(5-1) = 2,$ with a mid-line of $y = \frac{1}{2}(1+5) = 3,$ i.e. a vertical shift of 3. However, since the graph has a minimum at $x = 0,$ A must be negative. Hence,
$f(x) = -2\cos\left(\frac{3\pi}{4}x\right) + 3.$

5. a. *i.* The vertical intercept is fixed cost. It costs \$500,000 just to set up production.

ii. The value of *h* represents the number of units sold to make a maximum profit.

iii. Solving, we obtain approximately 4,724.75 units. Therefore 4,725 units must be sold.

b.

$$\begin{aligned} f(x) &= a(x-h)^2 + k \\ &= a(x-5)^2 + 7 \\ 5 &= a(3-5)^2 + 7 \to a = -0.5 \end{aligned}$$

Therefore, $f(x) = -0.5(x-5)^2 + 7.$

6. a. $f(w) = \dfrac{16+w}{100+w}$

b. $f(5)$ is the fraction of the team that is female when 5 women are added, which is $\frac{21}{105} = \frac{1}{5}.$

c. $w = f^{-1}(N) = \dfrac{100N - 16}{1-N}$

d. $f^{-1}(0.3)$ is the number of women that are added to the team so that three-tenths of the team is female (which for this team is the addition of 20 women.)

7. a. $36^\circ F$

b. $68^\circ F$

c.

$$\begin{aligned} 60 &= 68 - 32e^{-.08t} \\ -8 &= -32e^{-.08t} \\ \frac{1}{4} &= e^{-.08t} \\ t &= -\frac{\ln\left(\frac{1}{4}\right)}{0.8} \end{aligned}$$

d. From part (c), this time is approximately 1.73 hours after 8:00 am or about 9:44am