NCC Pre Calculus Partnership Program
Final Examination – Spring, 2010 Form A

Part I: Answer all 25 questions in this part. Each question is worth 2 points. Leave all answers in EXACT form, i.e., in terms of $e, \pi, \ln, \sqrt{\ }$, etc., unless otherwise instructed. No partial credit will be given.

1. Find the average rate of change for $y = x^2 - 3x$ between $x = -4$ and $x = 2$.

2. The cost, in dollars, of producing q thousand candy bars is given by $A = C(q)$. What are the units of $C^{-1}(A)$?
   a. thousands of dollars
   b. dollars per candy bar
   c. thousands of candy bars
   d. candy bars per dollar
   e. none of these

3. If $f(t) = -b\cos\left(\dfrac{\pi}{a}t\right) + 200$ then the period of $f(t)$ is

   a. 2π b. $2a$ c. $\dfrac{2}{a}$ d. $\dfrac{-b}{2a}$ e. $\dfrac{a}{2}$

4. Find the equation of the line that intersects the graph of $f(x) = x^2$ at $x = -2$ and $x = 3$.

5. The point (m,n) is on the graph of $y = f(x)$. If $k > 0$, then one point on the graph of $y = f(x) - k$ is
   a. $(m-k,n)$ b. $(m,n+k)$ c. $(m-k,n-k)$ d. $(m,n-k)$ e. none of these

6 . Find a possible formula for the polynomial of least degree whose y-intercept is 1, has a double zero at $x = -1$ and another zero at $x = 3$.

7. Find the slope of the line containing the points $\left(a, a^2\right)$ and $\left(a+h, (a+h)^2\right)$. Express your answer in simplest form.

8. A store is offering a special sale on plasma televisions. After reducing the original price of the television by \$120, an additional 15% discount is being offered. Write an equation that will represent the new cost, C, as a function of the original selling price, p.

9. If a population grows by 7% per year, then how many years, to the nearest tenth, will it take to triple?
   a. 15.1   b. 16.2   c. 17.9   d. 28.6   e. 42.9

10. The volume of the air, v, in cubic centimeters, in the lungs of a runner is modeled by the equation $v = k\sin(b\pi t) + h$, where t is in minutes. What is the minimum volume of air in the runner's lungs at any time? Assume all constants are positive.

11. Write an equation for a rational function $y = f(x)$, such that the graph of the function has a vertical asymptote at $x = 2$, a horizontal asymptote $y = 3$ and a zero at $x = -5$.

12. What value of x makes $\ln(\log x) = 0$ true?

13. Which one of the following is a true statement?
    a. If $g(f(1)) = 1$, then f and g are inverses of each other.
    b. All exponential functions are invertible.
    c. $g(f(x)) = f(g(x))$ for all functions f and g
    d. All linear functions are invertible.
    e. none of these

14. Solve for x: $3\log y = 2\log x$

15. Suppose $f(x) = 2b^x$. Let $(a, f(a))$ and $(c, f(c))$ be two points on the graph where $a < c$ and $f(a) > f(c)$. Which of the following is true?
    a. $b > 1$ b. $b < 1$ c. $0 < b < 1$ d. $b = 1$ e. can't be determined

16. What is the range of the function $f(x) = \pi - \left(\dfrac{\pi}{3}\right)^{-x}$?

    a. all real umbers   b. $y < \pi$ c. $y > \pi$ d. $y < \dfrac{2\pi}{3}$ e. $y < \dfrac{4\pi}{3}$

17. Air is put through a series of n filters to remove impurities. Each filter removes all but 20% of the remaining impurities Q, in the air. Which formula models this process? Assume Q_0 is the initial amount of impurities in the air.
    a. $Q = Q_0(0.2)^n$ b. $Q = Q_0(0.8)^n$ c. $Q = Q_0 e^{0.2n}$ d. $Q = Q_0 e^{-8n}$ e. none of these

18. Arrange the following functions in order from smallest to largest in terms of growth as x becomes infinitely large.
    A. $f(x) = 1000x + 10^6$ B. $f(x) = 10000\ln(x) + 2$ C. $f(x) = 1.2^x + 0.1$
    D. $f(x) = 100x^{100} - 2$

    a. A B D C
    b. B A C D
    c. A B C D
    d. D C A B
    e. B A D C

19. Assume the quantity y varies directly with x. If $y = f(x)$ is a linear function with a positive slope, then $x = f(y)$ is a linear function with a slope whose value is:

    a.   positive and the same as the slope of $y = f(x)$

    b.   positive and the reciprocal slope to the slope of $y = f(x)$

    c.   the negative of the slope of $y = f(x)$

    d.   the negative of the reciprocal of the slope of $y = f(x)$

    e.   none of these

20. Let $h = g(n)$ denote the number of hours it takes to read n novels. Let $n = f(c)$ denote the number of novels that have to be read for c classes. What does $g(f(c))$ tell you?

    a.   the number of novels that c classes read

    b.   the number of classes that read n novels

    c.   the number of hours of reading for c classes

    d.   the number of hours to read n novels

    e.   none of these

21. If k is a constant, what is the value of $f(e^k)$ if $f(x) = \begin{cases} 4 & \text{if } -7 \le x < -3 \\ 2 - x & \text{if } -3 \le x \le 0 \\ 3\ln x & \text{if } x > 0 \end{cases}$

    a. $3\ln k$ b. $2 - e^k$ c. 4   d. $3k$ e. can't be determined

22. Consider the function defined by the table below.

x	−3	−2	−1	0	1	2	3
$h(x)$	k	m	n	0	$-k$	$-m$	$-n$

If $x = -1$, the value of the function $(-h(-x + 2))^{\frac{1}{2}}$ is

    a. $\sqrt{n}$ b. $-\sqrt{n}$ c. $-\sqrt{-n}$ d. $\sqrt{k}$ e. none of these

23. If the functions $f(x)$ and $g(x)$ are defined by the table below and if $z(x) = g(f(x))$, then what is the value of $z^{-1}(3)$?

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	10	1	8	2	6	3	4	9	5	7
$g(x)$	7	8	9	10	1	2	3	4	5	6

24. The graph of $f(x)$ is shown below where the coordinates of point A are $(2, 3)$. If $h(x) = -f\left(\frac{1}{2}x\right) + 1$, what are the new coordinates of point A?
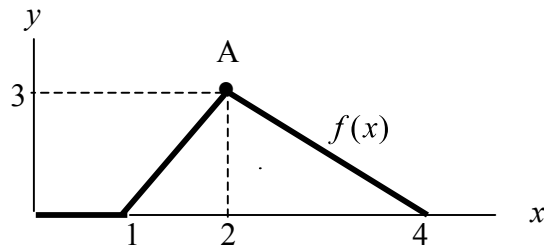

25. The graph shown below shows a function of the form $f(x) = ax + b + 2\cos(\pi x)$. Determine the value of ab (i.e. a times b). The graph is not drawn to scale.
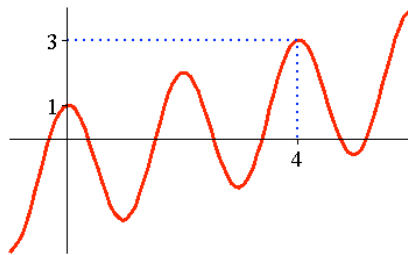
Part II: Before you begin, spend a few minutes reading each question. Answer *only* 5 *questions* in this part. Each question is worth 10 points. Be sure you clearly indicate the questions you *do not* wish to be graded. Partial credit will be awarded for work that is partially correct. Therefore, show all work in the blue book.

1. An auto rental agency charges its customers based *only* on the number of days the car is used and the number of miles driven. Suppose a patron rents a car for 3 days, drives it 218 miles, and is charged $196.67 (excluding sales tax.) Another patron rents the same type of car for 3 days, drives it 382 miles, and is charged $277.03 (excluding taxes). **(3, 1, 1, 3, 2)**

   a. Assuming the daily rate and the mileage charge for the car rentals are constant, find linear equation relating the total cost, $C(m)$, for renting this type of car for 3 days as a function of the number of miles driven, m. Round all constants to 2 decimal places.

   b. What is the *practical* meaning of the slope?

   c. What is the *practical* meaning of the vertical-intercept?

   d. Suppose I want to rent the same car for 3 days, but only have $160 to spend (excluding taxes.) What is the maximum number of miles I can drive, *without exceeding* my budgeted amount? *Round your answer to the nearest tenth of a mile.*

   e. In order to save money, I plan to make a 382 mile trip in 2 days. Excluding tax, what would this trip cost?

2. A marketing company has determined that the price, p, of its product and the quantity, q, that is sold, obey the demand equation $p = -\dfrac{1}{6}q + 100$, where $0 \le q \le 600$. **(2, 3, 3, 1, 1)**

   a. Revenue is defined to be the number of items sold multiplied by the price per item. Express the revenue R as a function of q.

   b. *Algebraically* determine the quantity that maximizes revenue?

   c. What is the maximum revenue?

   d. Which function, $R(q-50)$ or $R(q)-50$, will give a function that has the same maximum revenue?

   e. Which function, $R(q+100)$ or $R(q)+100$, will give a function where the quantity sold that produces the maximum revenue remains unchanged?

3a. The graphs of a linear function L and an exponential function G are shown below. Find a formula for $G(x)$. All answers should be in exact form. Note that the two functions intersect the y-axis at the same point. **(5)**
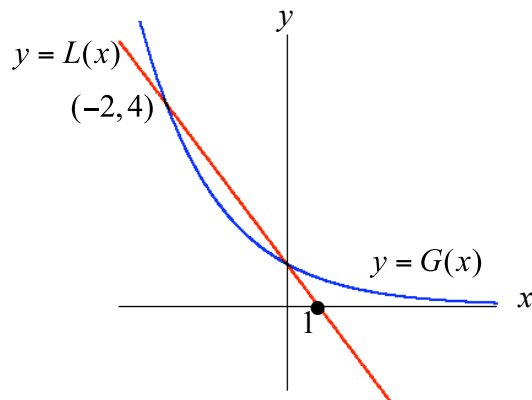

   b. A number of years ago, several people were injured on Black Friday, the day after Thanksgiving, when a large crowd tried to push through a store's doors before the doors were opened. Other stores have had problems even after the stores were opened due to large crowds.
   In order to head off any problems, a manager at the mall wants to estimate the number of customers on the day before Christmas. She collected data from the previous three years and determined that the crowds followed a general pattern. When the mall opens at 8 am, 500 people enter, and the total number of people in the mall doubles every 40 minutes. When the number of people in the mall reaches 10,000 she feels she must station security guards at the doors of the mall to control the crowds. How many minutes after the store opens should the guards be deployed? Round your answer to the nearest minute. **(5)**

4a. A weight is suspended from the ceiling by a spring. If we compress the spring and then release the weight, the distance, d, from the ceiling to the weight, measured in cm, can be modeled by $d(t) = 4\sin(2\pi t) + 10$, where the time, t, is measured in sec. The graph of the motion is shown below. For the graph below, find the four t values when $d = 12$. Only algebraic solutions will be accepted and all answers must be in *exact* form. **(4)**
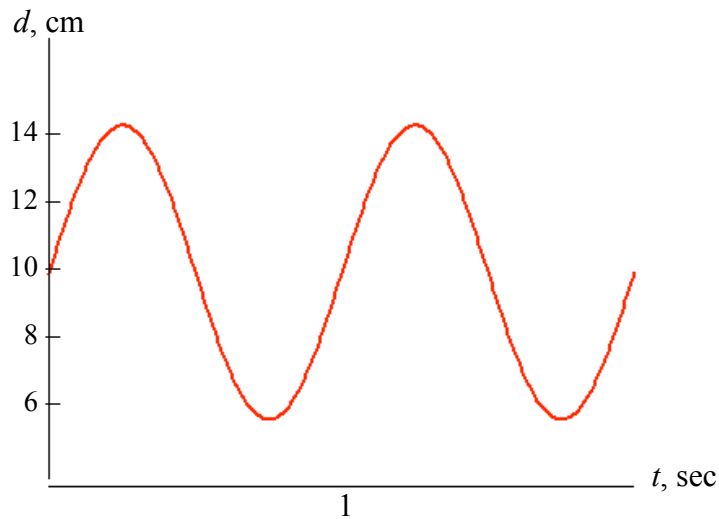

b. The graph below shows a periodic function tangent to the parabola $y = x^2 - 8x + 21$ at its vertex; i.e. the periodic function intersects the parabola only at its vertex. **(2, 4)**

   i. Use the method of completing the square to find the vertex of the parabola.
   ii. Express the periodic function in terms of cosine.
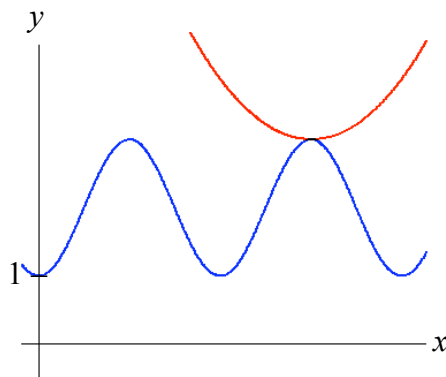
5a. A liquid form of penicillin is manufactured by a pharmaceutical firm. The profit, in dollars, the firm makes can be modeled by $P(x) = -0.003x^2 + 120x - 500,000$ where is x represents the number of units sold. **(2, 2, 2)**

    i. What does the vertical intercept mean *in practical terms* in this problem?

    ii. If $P(x)$ were to be put in the form $P(x) = a(x-h)^2 + k$, what does h represent *in terms of this problem*?

    iii. What is the least amount of units that must be sold for the firm to earn a profit? *Either an algebraic or graphical solution is acceptable.*

b. Ms. Smith leaves her house (located at the origin on the coordinate plane) and walks east in the positive x direction, holding an unopened umbrella. When she is five feet from home, it starts to rain and she opens her parabolic umbrella over her head. The peak of the umbrella is 7 feet above the ground and the coordinates of the peak are (5, 7). When opened, the bottom of the umbrella is two feet below the peak and the diameter of the umbrella at the bottom is 4 feet. See figure below. **(4)**
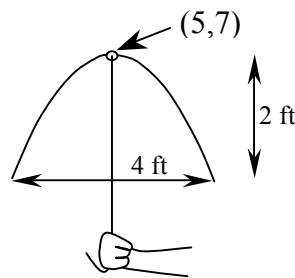
Write the equation for the umbrella curve, $f(x)$ in the form $f(x) = a(x-h)^2 + k$ by replacing a, h and k with their numerical values. Only an algebraic solution is acceptable.

6. A national team of scientists originally is composed of 16 women and 84 men. However, the committee forming this team is concerned about the lack of female representation and would like to add women to the team without removing any of the original male or female members. **(2, 2, 3, 3)**

    a. Let $N = f(w)$ be the fraction of the team that is female when w women have been added to the team. Find a formula for $f(w)$.

    b. Evaluate $f(5)$ and in one complete sentence, give its interpretation.

    c. Find a formula for $w = f^{-1}(N)$.

    d. Evaluate $f^{-1}(0.3)$ and in one complete sentence, give its interpretation.

7. Suppose a cold bottle of water is removed from a refrigerator at 8:00 a.m. It is then placed on the counter in a kitchen and left there. The temperature of the water, in degrees Fahrenheit t hours after 8:00 a.m. is given by $W(t)$, where $W(t) = 68 - 32e^{-0.8t}$. **(2, 2, 4, 2)**

    a. What is the temperature of the bottle of water when it was removed from the refrigerator?

    b. What is the temperature of the kitchen?

    c. *Using algebra*, find the time when the temperature of the water was $60°F$. Leave your answer in *exact* form. Be sure to show all work.

    d. Using your answer from part c, find to the nearest *minute,* the time of day at which this occurs.