An Outline of Topics and Concepts for Review[1]

Chapter One

- Function definition
  input and output
- Vertical line test
- Average rate of change
- Linear functions
- y changes at a constant rate
- Three forms of a linear function
  $y = b + mx$
  $y - y_0 = m(x - x_0)$
  $Ax + By + C = 0$
- Properties of linear functions
  Interpretation of slope and intercepts
  Solutions of equations
  Parallel lines: $m_1 = m_2$
  Perpendicular lines: $m_1 = -\frac{1}{m_2}$
- Fitting lines to data

Chapter Two

- Input and output
- Evaluating functions: finding $f(a)$ for a given a
- Solving equations: finding x if $f(x) = b$
- Domain and range
  Domain: the set of input values
  Range: the set of output values
- Inverse functions
  If $y = f(x)$ then $f^{-1}(y) = x$
  evaluating $f^{-1}(b)$ for a given value of b
  Interpretation of $f^{-1}(b)$
  Formula for $f^{-1}(y)$ given a formula for $f(x)$

[1]*Functions Modeling Change: A Preparation for Calculus*, 3th edition, by Connally, Hughes-Hallett, Gleason, et al.

Chapter Three

- Exponential functions
  Value of $f(t)$ changes at a constant *percent rate* with respect to *t*.

- General formulas for exponential functions
  $f(t) = ab^t$, $b > 0$
  $f(t)$ increasing if $b > 1$
  $f(t)$ decreasing if $0 < b < 1$
  Growth factor: $b = 1 + r$
  Growth rate: *r,* percent change as a decimal

- Comparing linear and exponential functions
  An increasing exponential function eventually becomes larger than any linear function.

- Graphs of exponential functions
  Concavity
  Asymptotes
  Effects of parameters *a* and *b*
  Finding equations of exponential functions
  Solving exponential equations graphically
  Fitting exponential functions to data

- The number *e*
  Continuous growth $f(t) = ae^{kt}$
  If $k > 0, f$ is increasing
  If $k < 0, f$ is decreasing
  The continuous growth rate is k

- Compound interest
  If you compound *n* times per year, then balance is $B = P\left(1 + \frac{r}{n}\right)^{nt}$
  Nominal rate versus effective rate

- Horizontal asymptotes and limits to infinity

Chapter Four

- Logarithms
  $y = \log x$ means $10^y = x$
  $\log 10 = 1$, $\log 1 = 0$
  $y = \ln x$ means $e^y = x$
  $\ln e = 1$, $\ln 1 = 0$

- Properties of logs
  $\log(ab) = \log a + \log b$
  $\log(a/b) = \log a - \log b$
  $\log b^t = t \log b$
  $\log 10^x = 10^{\log x} = x$
  $\log_e e^x = e^{\ln x} = x$

- Converting between base *b* and base *e*
  If
  $Q = ab^t$ and $Q = ae^{kt}$, then $k = \ln b$, or

- Solving equations using logs

- Logarithmic functions
  Graph
  Domain
  Range
  Concavity
  Asymptotes

- Applications of logarithms
  Half life

- Doubling timems
  Half life
  Doubling time

Chapter Five

- Vertical shifts: $y = f(x) + k$
  Upward if $k > 0$
  Downward if $k < 0$

- Horizontal Shifts: $y = f(x+k)$
  Left if $k > 0$
  Right if $k < 0$

- Reflections
  Across the x-axis: $y = -f(x)$
  Across the y-axis: $y = f(-x)$

- Symmetry
  About the y-axis: $f(-x) = f(x)$: this is an even function
  About the origin: $f(-x) = -f(x)$: this is an odd function

- Stretches and compressions
  Vertical: $y = kf(x)$:
  If $k > 0$, it is a stretch
  If $k < 0$, it is a compression
  Horizontal: $y = f(kx)$:
  If $k > 0$, it is a compression
  If $0 < k < 1$, it is a stretch
  If $k < 0$, it is a reflection across the y-axis

- Quadratic functions
  Standard form: $f(x) = ax^2 + bx + c$
  Vertex form: $f(x) = a(x-h)^2 + k$
  If $a > 0$, it opens upward
  If $a < 0$, it opens downward
  Vertex is at (h,k)
  Axis of symmetry at $x = h$

- Maximum value

- Minimum value

- Completing the square

Chapter Six

- Periodic functions
  Period: the smallest c such that $f(x+c) = f(x)$

- Sine and Cosine
  Angular rotation correspond to points on a circle of radius r
  $x = r\cos\theta$
  $y = r\sin\theta$

- Radians
  Definition
  2π radians $= 360$ degrees
  Arc length: $s = r\theta$

- Sinusoidal Functions
  $f(x) = A\sin\left(B(t-h)\right) + k$
  $f(x) = A\cos\left(B(t-h)\right) + k$
  Amplitude $= |A|$
  Period $= \dfrac{2\pi}{|B|}$
  Midline: $y = k$
  Phase shift: Bh
  Horizontal shift: h

- Other Trigonometric functions
  Tangent
  Secant
  Cosecant
  Cotangent

- Identities
  $\sin^2\theta + \cos^2\theta = 1,\ \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

- Inverse trigonometric functions
  $\sin^{-1} y = t$ means $y = \sin t, -\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$
  $\cos^{-1} y = t$ means $y = \cos t,\ 0 \le t \le \pi$
  $\tan^{-1} y = t$ means $y = \tan t,\ -\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$

- Reference angles

- Solving trigonometric equations

Chapter Eight

- Composition of functions
  - Notation: $h(x) = f\left(g(x)\right)$
  - Domain and range: Decomposition
- Inverse functions
  - Definition:
    - $f^{-1}(Q) = t$ if and only if $Q = f(t)$
  - Invertibility: horizontal line test
  - Domain and range
  - Restricting domain of a function to construct an inverse
- Combinations of functions
  - Sums
  - Differences
  - Products
  - Quotients

Chapter Nine

- Proportionality
  - Direct
  - Indirect
- Power functions
- Polynomials
  - $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$
  - All terms have non-negative, integer exponents
  - Long-run behavior is like $y = a_n x^n$
  - Short-run behavior:
    - Zeros correspond to factors
    - Multiple zeros
- Rational functions
  - Ratio of polynomials: $r(x) = \dfrac{p(x)}{q(x)}$
  - Long-run behavior: Horizontal asymptote is ratio of highest power terms
  - Short-run behavior: Vertical asymptote at zeros of $q(x)$, provided $p(x) \neq 0$
  - Zeros of $r(x)$ are at $p(x)$, provided $q(x) \neq 0$
- Using limits to understand short- and long-run behavior
- Rational functions
  - Ratio of polynomials: $r(x) = \dfrac{p(x)}{q(x)}$
  - Long-run behavior: Horizontal asymptote is ratio of highest power terms
  - Short-run behavior: Vertical asymptote at zeros of $q(x)$, provided $p(x) \neq 0$
  - Zeros of $r(x)$ are at $p(x)$, provided $q(x) \neq 0$
  - Using limits to understand short- and long-run behavior

An Outline of Topics and Concepts for Review