Chapter 9 - Interval Estimation

Section 9.4: Interval Estimation: Confidence Intervals for the Population Mean

Distribution of sample means.

We answer the question... When I take a sample to estimate the population mean, how close am I?

\[ \mu_X = \mu \]
\[ \sigma_X = \frac{\sigma}{\sqrt{n}} \]

68% of the area is within (\( \mu \pm 1\sigma_X \))
95% of the area is within (\( \mu \pm 2\sigma_X \))
99.7% of the area is within (\( \mu \pm 3\sigma_X \))

Estimation

Point Estimate:
A single number which uses sample information to estimate the value of a population parameter.

Interval Estimate:
An estimate of the range of values within which the population parameter is likely to fall.

Consider a population of 20-year old men and women.
Blood Pressure: \( \mu = 120 \) \( \sigma = 20 \)
Take repeated samples of size \( n = 100 \)
Distribution of sample means is normal

Recall: For a normal distribution, 90% of the data falls between \( z = -1.64 \) and \( z = +1.64 \).
Confidence Intervals for the Population Mean

Consider a population of 20-year old men and women. Blood Pressure: $\mu = 120$, $\sigma = 20$
Take repeated samples of size $n = 100$
Distribution of sample means is normal

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2$$

Take repeated samples of $n=100$ from this population and plot the sample means.

Then extend a range of 1.64$\sigma$ above and below each sample mean.

This range of values is called the 90% CONFIDENCE INTERVAL

For 90% of the sample means, the range extended about the sample mean will include the true mean of the population, $\mu$.
Thus 90% is the CONFIDENCE LEVEL.

To determine the population mean, $\mu$, at a 90% Confidence Level, take a sample and determine its mean, $\bar{x}$.

There is a 90% probability that $\mu$ will be in the range
$$\bar{x} - (1.64)(SE) < \mu < \bar{x} + (1.64)(SE)$$

Other Confidence Levels

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>z-score</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>1.64</td>
<td>$\bar{x} \pm 1.64 \frac{\sigma}{\sqrt{n}}$</td>
</tr>
<tr>
<td>95%</td>
<td>1.96</td>
<td>$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$</td>
</tr>
<tr>
<td>99%</td>
<td>2.58</td>
<td>$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$</td>
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</tbody>
</table>

Margin of Error
To estimate the mean amount overdue in all its delinquent accounts, a bank randomly samples 49 accounts and finds the sample mean, $\bar{x}$, to be $237.

Based on past history, the bank uses a standard deviation, $\sigma$, of $53.$

1. Determine the 95% Confidence Interval for the mean amount overdue.

2. Determine the Margin of Error.

For a 95% Confidence Level, $z = \text{InvNormal}(0.975) = 1.96$

Since $\sigma = 53$ and $n = 49,$

the Standard Error of the Mean = $\frac{\sigma}{\sqrt{n}} = \frac{53}{\sqrt{49}} = 7.57$

Since the Sample Mean was $237,$ the 95% Confidence Interval is

$\bar{x} - z \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z \cdot \frac{\sigma}{\sqrt{n}}$

$237 - (1.96)(7.57) < \mu < 237 + (1.96)(7.57)$

$222.16 < \mu < 251.84$

Margin of Error = $251.84 - 237 = 14.84$

Distribution of sample means. $\sigma$ is unknown

$t$ Distribution

$s$ is the sample standard deviation (determined from the sample)

To find, say the 90% Confidence Interval, determine the $t$-score which encloses 90% of the sample means.

Once the $t$-score ($t_{95\%}$) is found,

$\bar{x} - t_{95\%} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{95\%} \frac{s}{\sqrt{n}}$ with 90% probability

Distribution of sample means. $\sigma$ is unknown

To find the $t$-scores when you know the area under the $t$-distribution, you must use a “$t$-table.”

For example, to know the $t$-scores that enclose 90% of the area in the center of the $t$-distribution using a sample size of $n = 10,$
Section 9.4: Interval Estimation

Confidence Intervals for the Population Mean

*To find the t-score when you know the area under the t-distribution, you must use a “t-table.”*

Degrees of Freedom (df) is simply $(n - 1)$

...for $n = 10$, $df = 9$  $t = 1.83$

To find the t-score when you know the area under the t-distribution, you must use a “t-table.”

Degrees of Freedom (df) is simply $(n - 1)$

For $n = 10$, $df = 9$  $t = 1.83$

### Example 8.3

An English professor estimates the number of typing errors per page in term papers by collecting a sample of 36 papers. From this sample, she finds a mean of 4.6 errors per page with $s = 1.4$. What is the 90% confidence interval for all term papers?

**Sample Std Dev**  
$s = 1.4$

**Mean Errors/Page**  
$x = 4.6$

**Sample Size**  
$n = 36$

$df = 36 - 1 = 35$

**Confidence Level**  
90%

**Confidence Interval**

For 90% Confidence Interval,

$$x - t_{95\%} \frac{s}{\sqrt{n}} < \mu < x + t_{95\%} \frac{s}{\sqrt{n}}$$

$$4.206 \text{ to } 4.994$$

**Sample mean**  
$x = 4.6$

**Margin of Error**  
$E = 0.394$
Chapter 9 - Interval Estimation

Interval Estimation Using the Calculator

To estimate the Summer traffic across the Throgs Neck Bridge, the DOT sampled the traffic on 10 randomly selected Summer days. In thousands of cars, the results were...

112 115 139 128 130
122 145 132 102 136

thousand cars per day.

Use the calculator to estimate the mean daily traffic and the Margin of Error for the daily Summer traffic at a 90%, 95% and 99% Confidence Level.

STATS ➤ TESTS ➤ TInterval

<table>
<thead>
<tr>
<th>List</th>
<th>C-Level</th>
<th>Calculate</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>.9</td>
<td></td>
</tr>
</tbody>
</table>

At a 90% Confidence Level

Est Mean Traffic = 126.1
Margin of Error = 7.72

Section 9.5: Interval Estimation: Population Proportion

In the population there are millions of Republicans, Democrats and Others.

The proportion of Republicans in the population is

\[ p_R = \frac{\text{Number of Republicans in the population}}{\text{Total population}} = \frac{N_R}{N} \]

If we take a sample of \(n\) people from the population, the proportion of Republicans in the sample will be

\[ \hat{p}_R = \frac{\text{Number of Republicans in the sample}}{\text{Sample size}} = \frac{x}{n} \]

The mean of the distribution of the proportion is \(p\) and the standard error of the proportion is

\[ \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \]

The proportion of Republicans in the population is

\[ p_R = \frac{\text{Number of Republicans in the population}}{\text{Total population}} = \frac{N_R}{N} \]

If we took repeated samples from the population and determined the proportions from each sample, they would form a normal distribution.

The mean of the distribution of the proportion is \(p\) and the standard error of the proportion is

\[ \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \]

When we are estimating the population proportion, \(p\), from the sample proportion, \(\hat{p}\), we use \(s_{\hat{p}}\) to represent the standard error of the proportion.

\[ s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]

Therefore, the confidence interval of a proportion is

90%: \( \hat{p} - (1.65)s_{\hat{p}} < p < \hat{p} + (1.65)s_{\hat{p}} \)

95%: \( \hat{p} - (1.96)s_{\hat{p}} < p < \hat{p} + (1.96)s_{\hat{p}} \)

99%: \( \hat{p} - (2.58)s_{\hat{p}} < p < \hat{p} + (2.58)s_{\hat{p}} \)
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Section 9.5: Interval Estimation: Population Proportion

From a semester’s class survey of 35 students, 30 students indicated that they believed in God.

From this sample what is the 90% confidence interval for the proportion of all students at Nassau who believe in God.

From the sample:

Sample proportion: \( \hat{p} = \frac{30}{35} = 0.857 \)

Standard error of the proportion: \( s_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.857(1-0.857)}{35}} \approx 0.059 \)

Use Normal Distribution for Confidence Intervals

90%: \( \hat{p} - (1.65)s_p < p < \hat{p} + (1.65)s_p \)

95%: \( \hat{p} - (1.96)s_p < p < \hat{p} + (1.96)s_p \)

99%: \( \hat{p} - (2.56)s_p < p < \hat{p} + (2.56)s_p \)

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Section 9.6: Determining Sample Size and Margin of Error

To estimate the mean flight time between two cities, a sample of 64 flights during the year was taken. The sample had a mean of 2 hours and a sample standard deviation, \( s \), of 20 minutes.

At a confidence level of 95%, what is the confidence interval for all flight times between those cities? What is the margin of error?

Since \( \sigma \) is unknown, we use \( T\text{Interval} \) with …

\[ \bar{x} = 120 \text{ minutes}, \]
\[ s_x = 20 \text{ minutes}, \]
\[ n = 64 \]
\[ \text{and C-Level} = .95 \]

The \( T\text{Interval} \) is \( (115, 125) \) minutes.

So the Margin of Error
\[ E = (125 - 115) / 2 = 5 \text{ minutes} \]
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Proportion Estimation Using the Calculator

To estimate a population proportion, \( p \), from a sample \( \hat{p} \),
use

\[ \text{STAT} \rightarrow \text{TESTS} \rightarrow 1\text{-PropZInt} \]

For example in a sample of \( n = 100 \) voters, \( x = 20 \) voters said
they would vote the Independent candidate. At a 95% Confidence level, what proportion of the population of voters are expected to vote Independent?

\[ 1\text{-PropZInt} \]
\[ \hat{p} = \frac{x}{n} = \frac{20}{100} = 0.2 \]
\[ C\text{-Level: .95} \]
\[ \text{Calculate} \]

\[ (0.1216, 0.2784) \]

Thus the 95% confidence interval for the proportion of voters who we expect will vote Independent is between 0.1216 and 0.2784.

0.1216 < 0.2 < 0.2784

0.2784 - 0.2 = 0.0784 = Margin of Error

Estimation Using the Calculator

Summary

<table>
<thead>
<tr>
<th>Estimate of the Population Mean (( \mu ))</th>
<th>Estimate of the Population Proportion (( p ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma ) known</td>
<td>( \sigma ) unknown</td>
</tr>
<tr>
<td>ZInterval (stats)</td>
<td>TInterval (stats)</td>
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<tr>
<td>: pop std dev</td>
<td>: sample std dev</td>
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<td>: sample size</td>
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<td>: sample mean</td>
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</tr>
<tr>
<td>C-level: Confidence level</td>
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